Modulation

Amplitude Modulation (AM)

Consider a carrier signal:

$$\boxed{v(t) = A(t)\cos(\omega_c t)}$$

In which the amplitude A(T) varies slowly with respect to the carrier frequency. Let:

$$A(t) = V_0\left(1 + m\cos(\omega_m t)\right)$$

modulation index

modulation frequency

Amplitude Modulation (AM)

$$\boxed{v(t) = V_0\left(1 + m\cos(\omega_m t)\right)\cos(\omega_c t)}$$

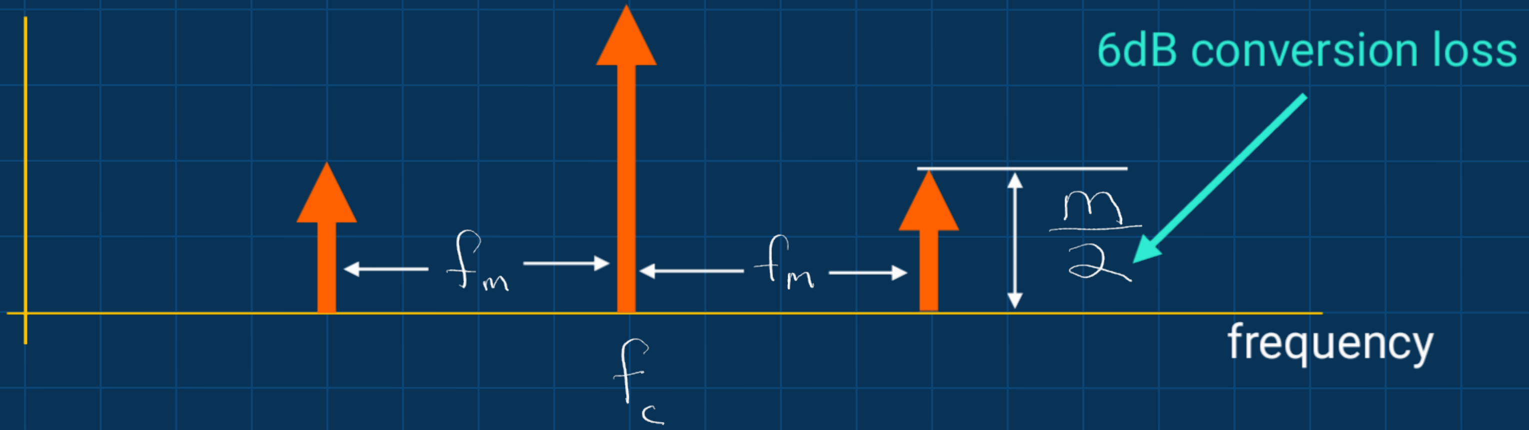
Most important trig identity in RF

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos(a)\cos(b) = \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b)$$

Amplitude Modulation (AM)

$$
\begin{aligned}
v(t) = {} & V_0 \cos(\omega_c t) && \text{carrier} \\
& + \frac{mV_0}{2} \cos((\omega_c + \omega_m)t) && \text{upper sideband} \\
& + \frac{mV_0}{2} \cos((\omega_c - \omega_m)t) && \text{lower sideband}
\end{aligned}
$$



Frequency Modulation

$$\boxed{\omega = \omega_c + \Delta\omega_m \cos(\omega_m t)}$$

$\omega_c =$ carrier frequency

ω_m modulation frequency

$\Delta\omega_m$ modulation amplitude

$$\omega = \frac{d\varphi}{dt}$$

$\varphi =$ RF phase

$$\varphi = \int \omega(t)\, dt$$

Frequency Modulation

$$\phi = \omega_c t + \frac{\Delta\omega_m}{\omega_m}\sin(\omega_m t)$$

$$v(t) = V_0 \cos\left(\omega_c t + \frac{\Delta\omega_m}{\omega_m}\sin(\omega_m t)\right)$$

yikes!

$$e^{j\alpha} = \cos(\alpha) + j\sin(\alpha)$$

$$\cos(\alpha) = \mathrm{Re}\{e^{j\alpha}\}$$

$$v(t) = V_0\,\mathrm{Re}\left\{e^{j\omega_c t}\, e^{j\frac{\Delta\omega_m}{\omega_m}\sin(\omega_m t)}\right\}$$

Frequency Modulation

MAX IV RF

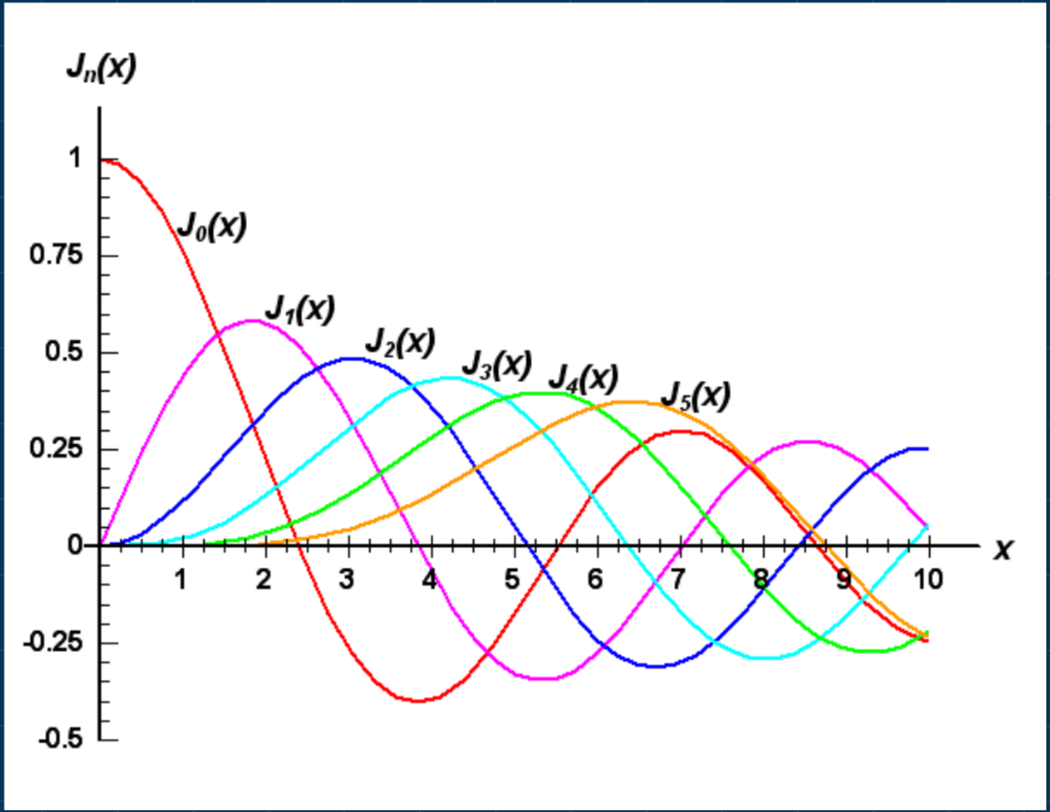
Bessel Identity to the rescue!

$$e^{jz\sin(x)} = \sum_{n=-\infty}^{\infty} J_n(x)\, e^{jnx}$$

$$v(t) = V_0 \,\mathrm{Re}\left\{ \sum_{n=-\infty}^{\infty} J_n\left(\frac{\Delta\omega_m}{\omega_m}\right) e^{j(\omega_c + n\omega_m)t} \right\}$$

$$v(t) = V_0 \sum_{n=-\infty}^{\infty} J_n\left(\frac{\Delta\omega_m}{\omega_m}\right) \cos((\omega_c + n\omega_m)t)$$

Bessel Functions



Frequency Modulation Spectrum