Introduction to RF for Particle Accelerators
Part 2: RF Cavities

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RF Cavity Topics

- Modes
  - Symmetry
  - Boundaries
  - Degeneracy
- RLC model
- Coupling
  - Inductive
  - Capacitive
  - Measuring
- Q
  - Unloaded Q
  - Loaded Q
  - Q Measurements
- Impedance Measurements
- Power Amplifiers
  - Class of operation
  - Tetrodes
  - Klystrons
- Beam Loading
  - De-tuning
  - Fundamental
  - Transient

RF and Circular Accelerators

For circular accelerators, the beam can only be accelerated by a time-varying (RF) electromagnetic field.

Faraday's Law

$$\oint_C \vec{E} \bullet d\vec{l} = -\frac{\partial}{\partial t} \iint_S \vec{B} \bullet d\vec{S}$$

The integral

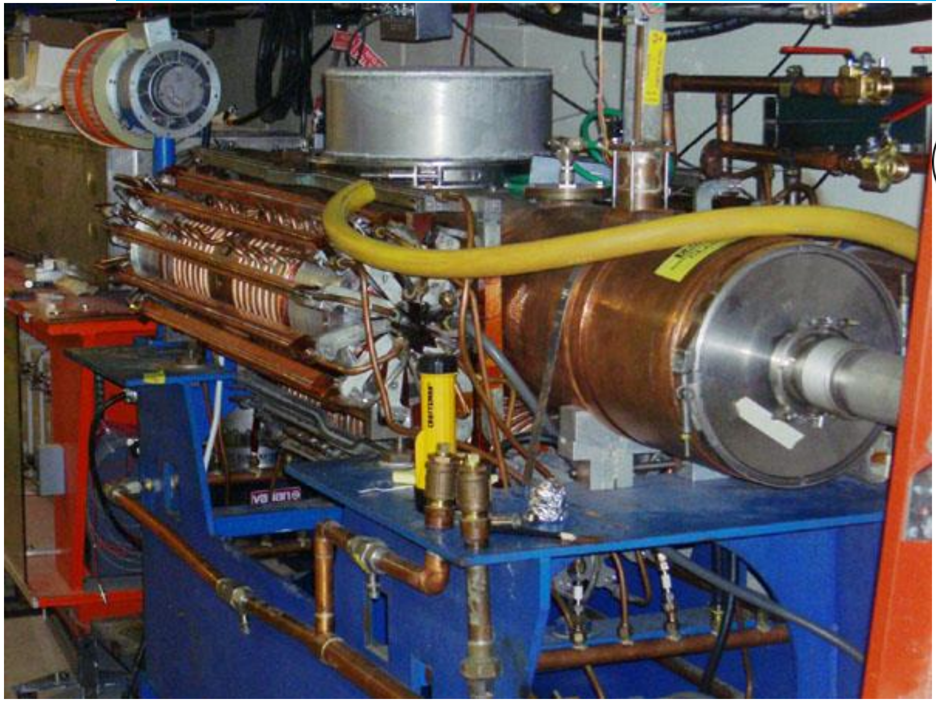
$$q\oint_C \vec{E} \bullet d\vec{l}$$

is the energy gained by a particle with charge q during one trip around the accelerator.

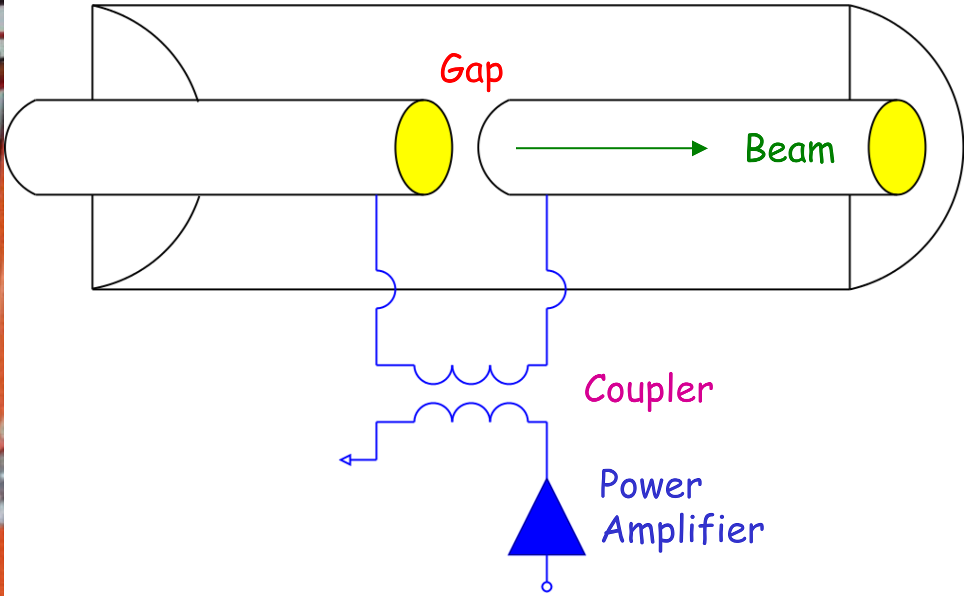
For a machine with a fixed closed path such as a synchrotron, if

$$\frac{\partial \vec{B}}{\partial t} = 0 \quad \text{then} \quad q\oint_C \vec{E} \bullet d\vec{l} = 0$$

RF Cavities



New FNAL Booster Cavity

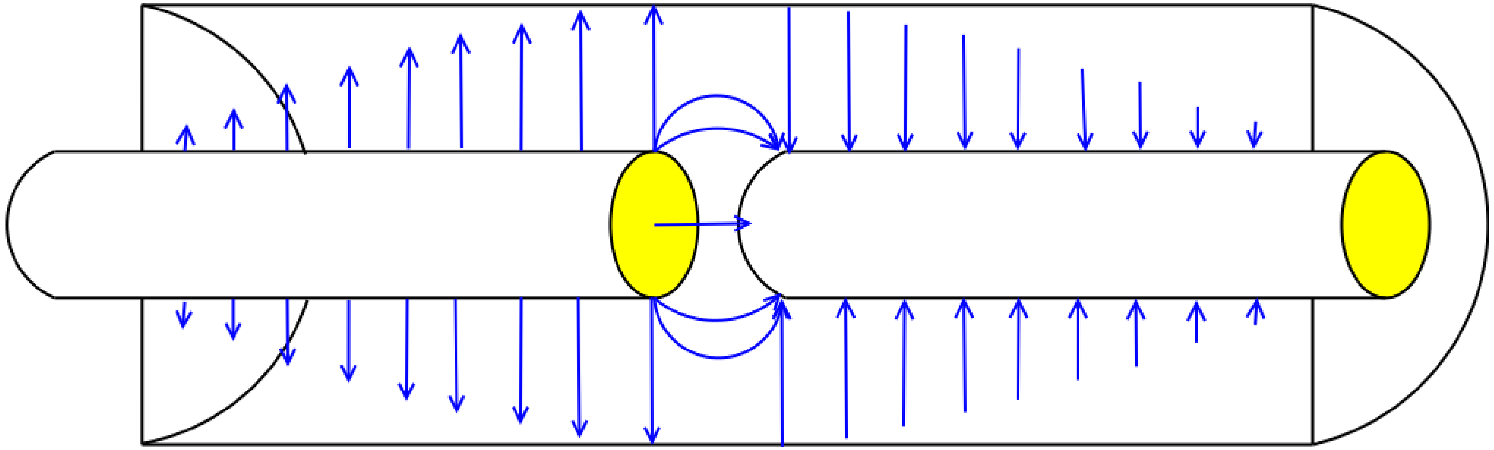


Transmission Line Cavity

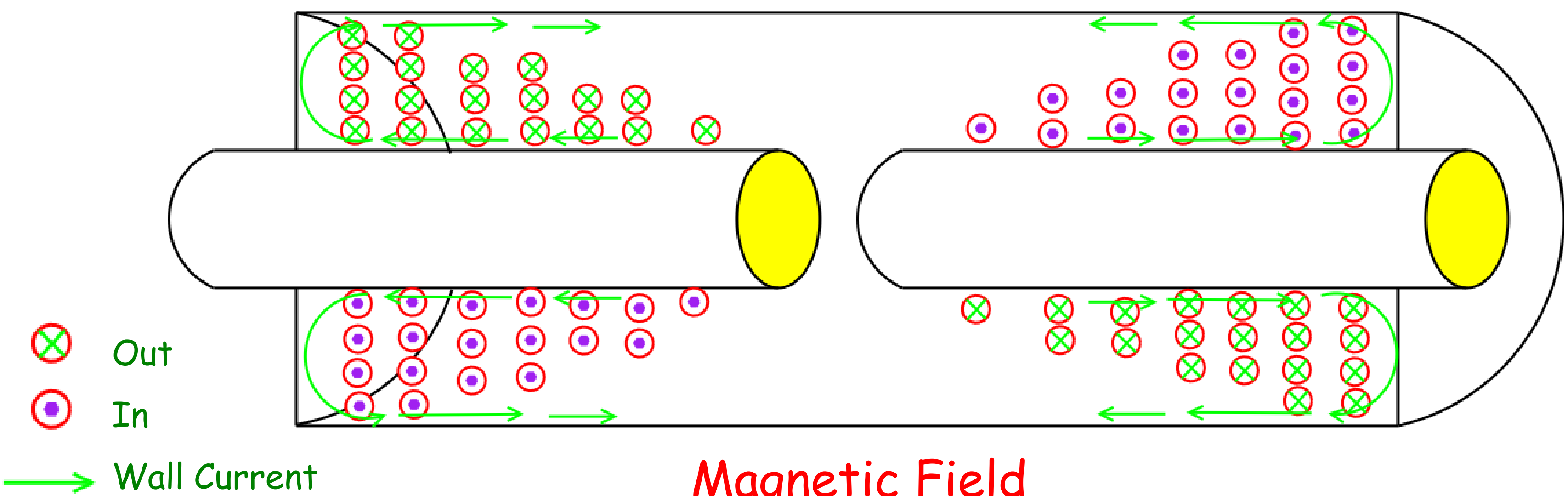
Multi-cell superconducting RF cavity

Cavity Field Pattern

For the fundamental mode at one instant in time:



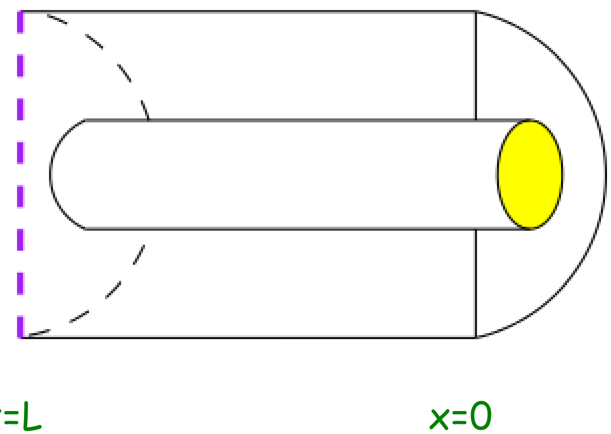
Electric Field



Magnetic Field

Cavity Modes

We need to solve only ½ of the problem



For starters, ignore the gap capacitance.

The cavity looks like a shorted section of transmission line

$$V = V^+ e^{-j\beta x} + V^- e^{+j\beta x}$$

$$Z_o I = V^+ e^{-j\beta x} - V^- e^{+j\beta x}$$

where Z_o is the characteristic impedance of the transmission line structure of the cavity

Cavity Boundary Conditions

Boundary Condition 1:

At x=0: V=0

$$V = V_o \sin(\beta x)$$

$$Z_o I = -jV_o \cos(\beta x)$$

Boundary Condition 2:

At x=L: I=0

$$\cos(\beta L) = 0$$

$$\beta_n L = (2n+1)\frac{\pi}{2}$$

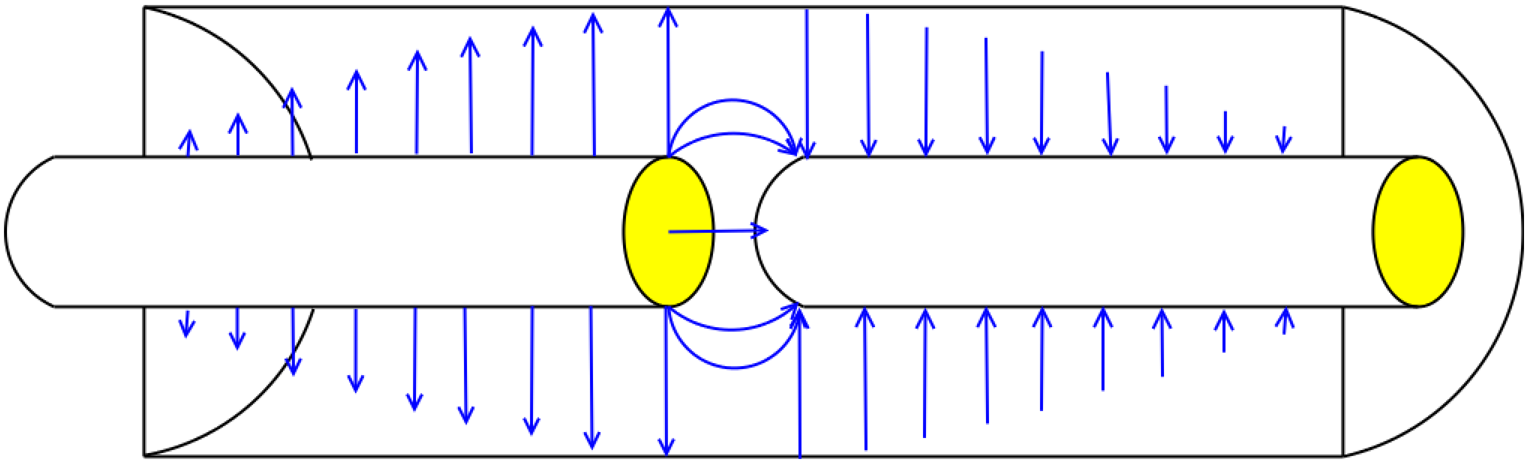
$$n = 0,1,2,3...$$

$$f_n = (2n+1)\frac{c}{4L}$$

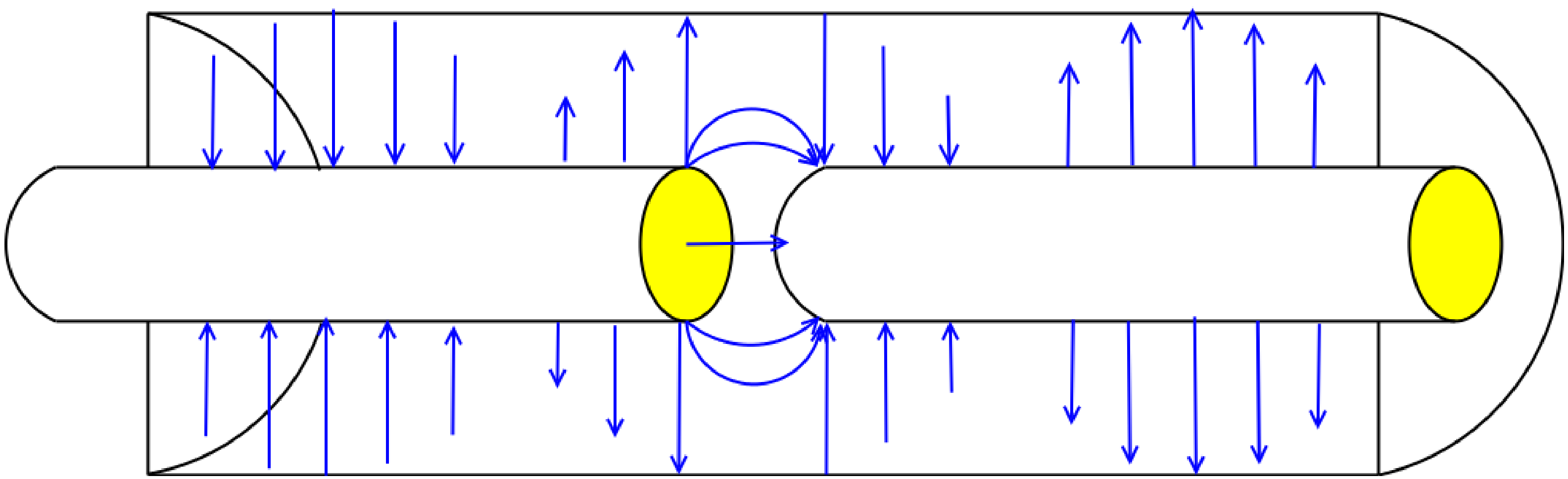
$$L = (2n+1)\frac{\lambda_n}{4}$$

Different values of n are called modes. The lowest value of n is usually called the fundamental mode

Cavity Modes

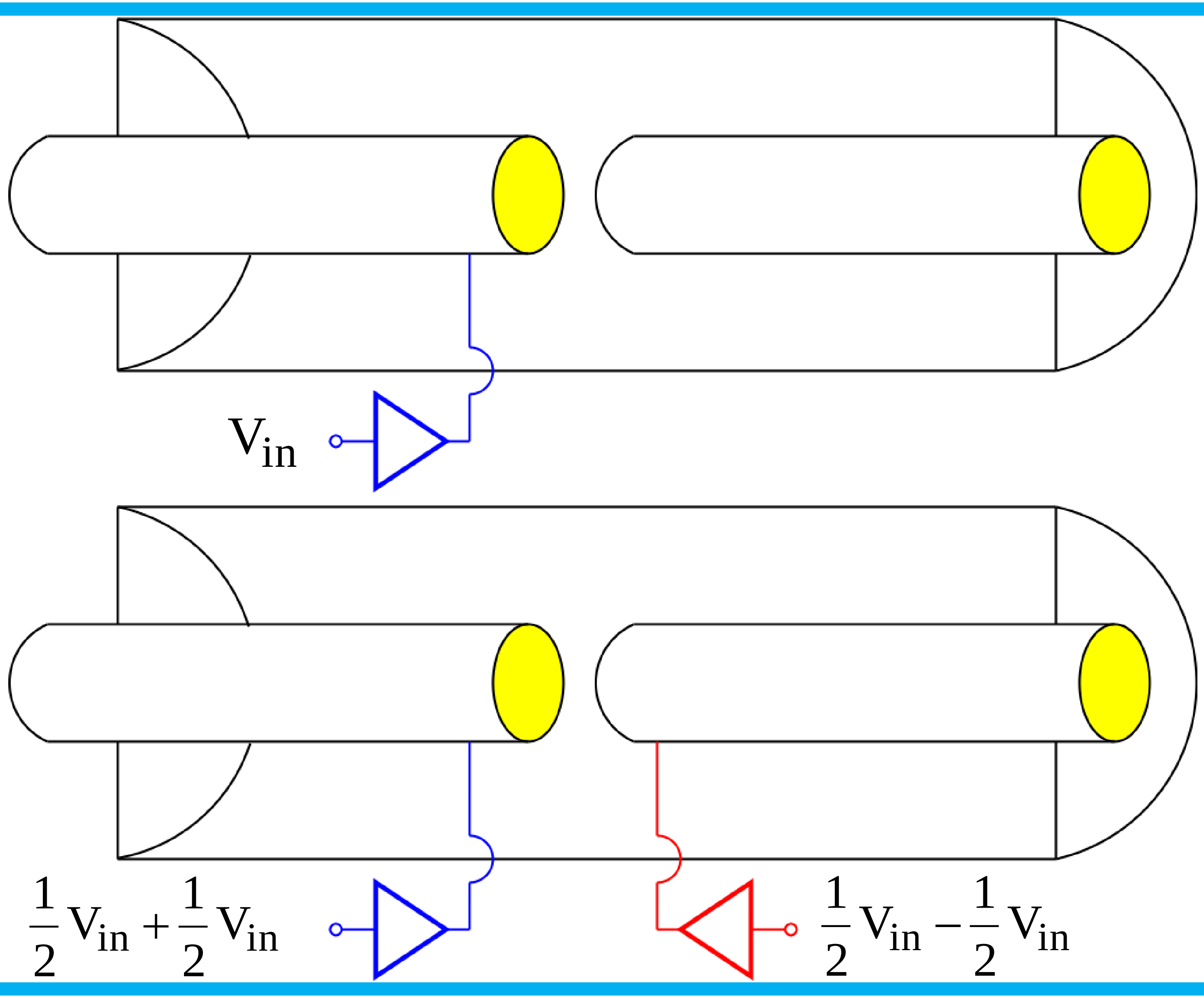


n=0

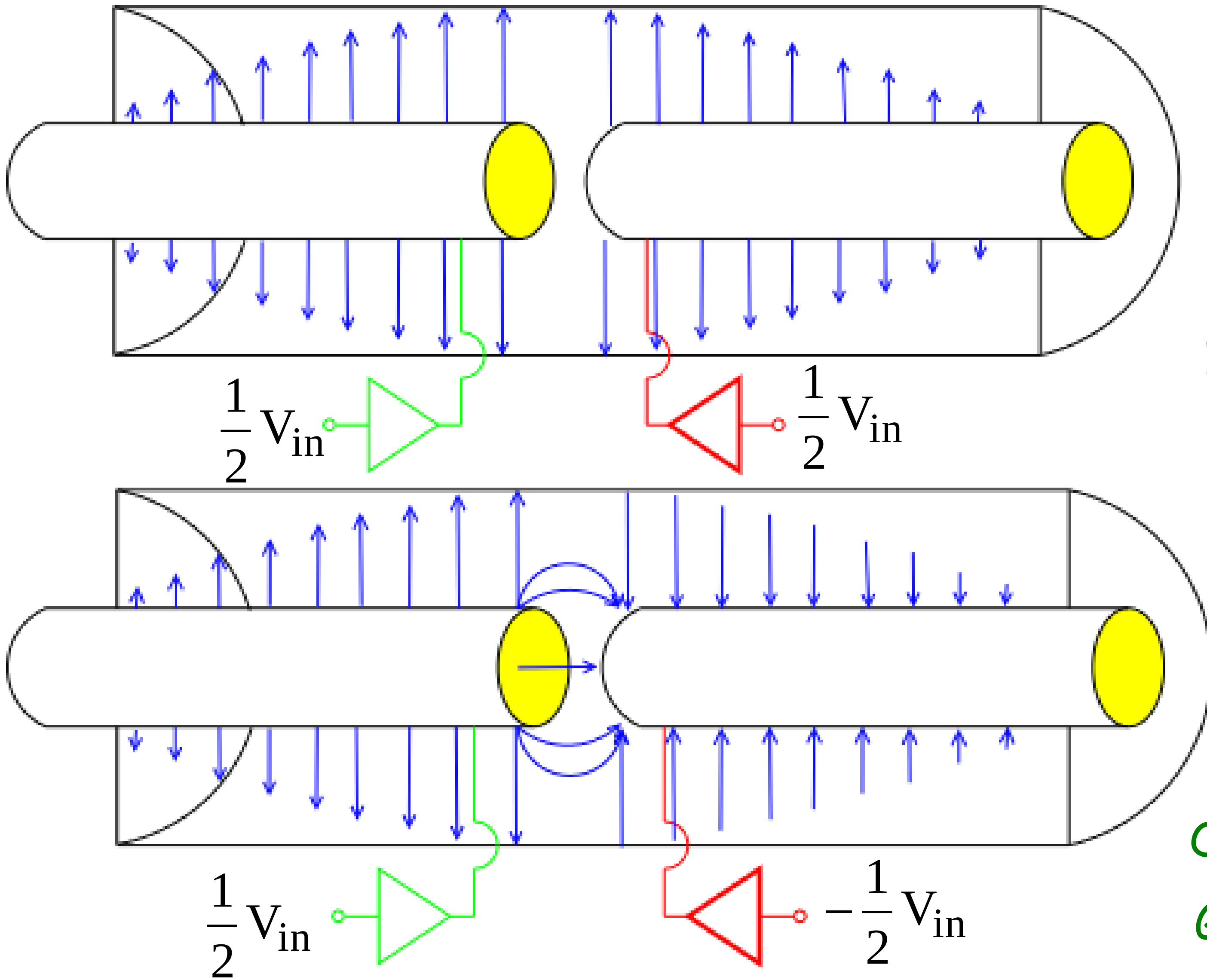


n=1

Even and Odd Mode De-Composition



Even and Odd Mode De-Composition



Even Mode

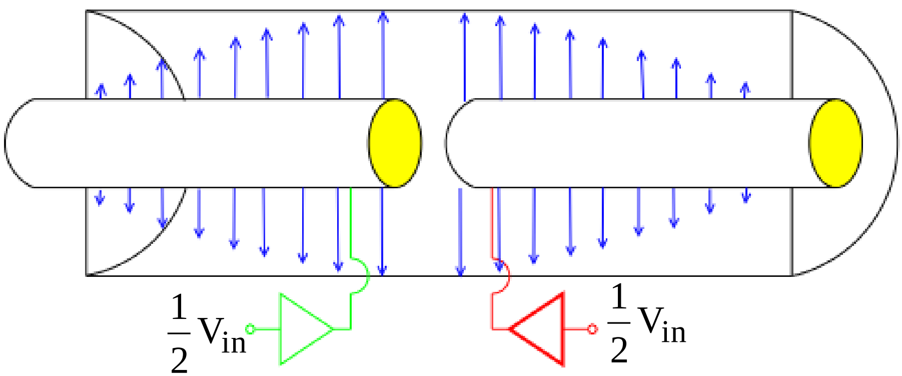
No Gap Field

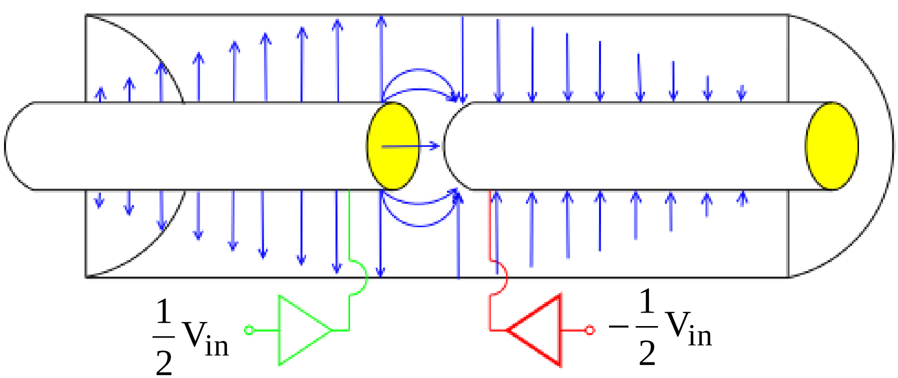
Odd Mode

Gap Field!

Degenerate Modes

- The even and odd decompositions have the <u>same</u> mode frequencies.
- Modes that occur at the same frequency are called degenerate.
- The even and odd modes can be split if we include the gap capacitance.
- In the even mode, since the voltage is the same on both sides of the gap, no capacitive current can flow across the gap.
- In the odd mode, there is a voltage difference across the gap, so capacitive current will flow across the gap.





Gap Capacitance

Boundary Condition 1:

At x=0: V=0

$$V = V_o \sin(\beta x)$$

$$Z_o I = -jV_o \cos(\beta x)$$

Boundary Condition 2:

At x=L: $I = j\omega C_g V$ where C_g is the gap capacitance

$$\omega C_g Z_o = \frac{\cos(\beta L)}{\sin(\beta L)}$$

RF Cavity Modes

Consider the first mode only (n=0) and a very small gap capacitance.

$$\beta L = \frac{\pi}{2} + \delta$$

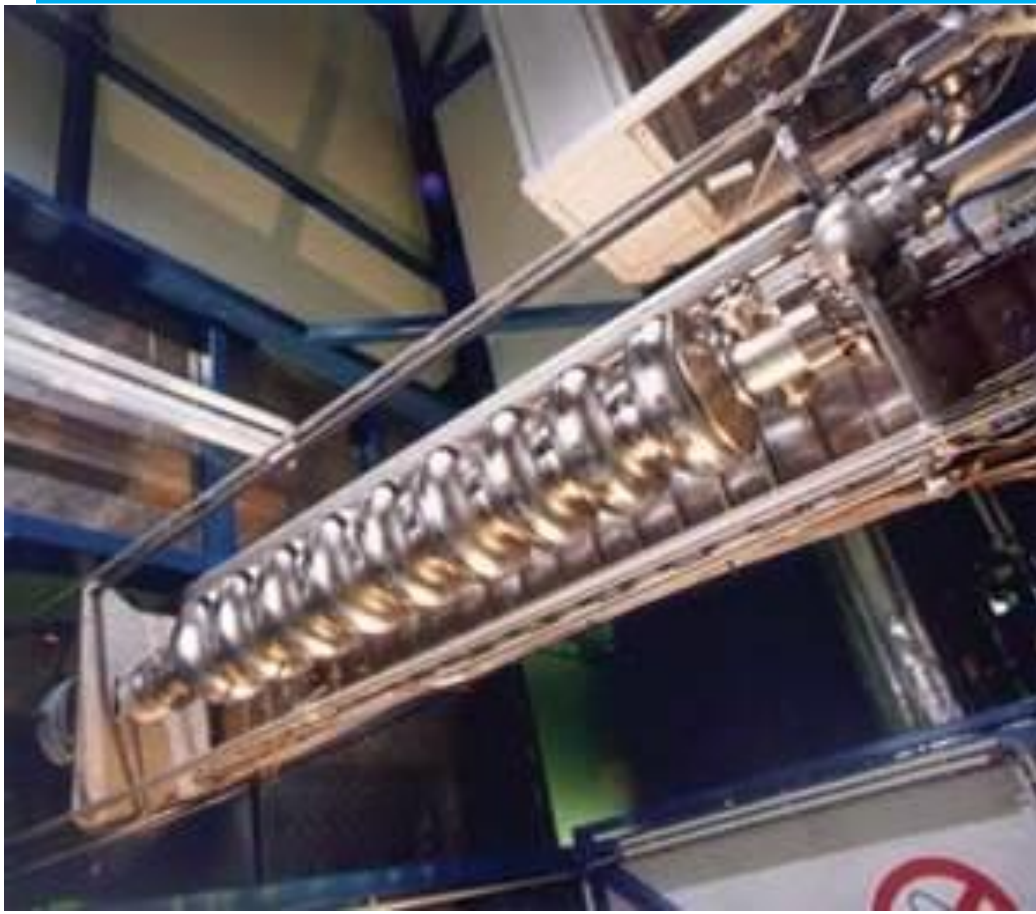
$$\frac{\cos(\beta L)}{\sin(\beta L)} \approx -\delta$$

$$\delta = -\omega_o C_g Z_o$$

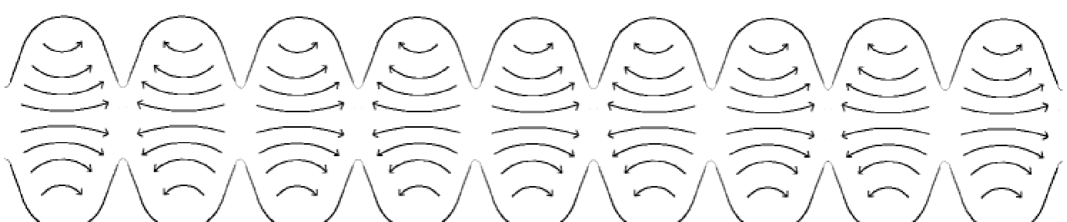
$$\frac{\Delta\omega_o}{\omega_o} = \frac{2}{\pi}\delta$$

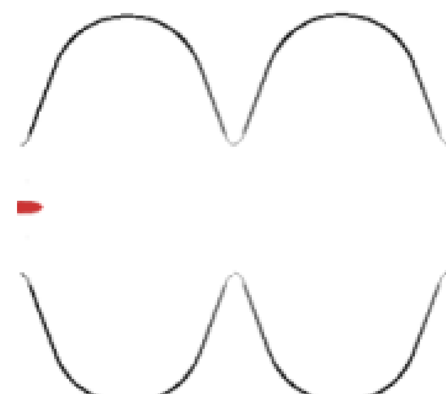
The gap capacitance shifts the odd mode down in frequency and leaves the even mode frequency unchanged

Multi-Celled Cavities



- Each cell has its own resonant frequency
- For n cells there will be n degenerate modes
- The cavity to cavity coupling splits these n degenerate modes.
- The correct accelerating mode must be picked





Cavity Q

- If the cavity walls are lossless, then the boundary conditions for a given mode can only be satisfied at a single frequency.
- If the cavity walls have some loss, then the boundary conditions can be satisfied over a range of frequencies.
- The cavity Q factor is a convenient way the power lost in a cavity.
- The Q factor is defined as:

$$Q = \frac{W_{\mathrm{stored}}}{W_{\mathrm{lost/cycle}}}$$

$$= \omega_o \frac{W_E + W_H}{P_L}$$

Transmission Line Cavity Q

We will use the fundamental mode of the transmission line cavity as an example of how to calculate the cavity Q.

Electric Energy:

$$W_E = \frac{1}{4}\iiint_{vol} \varepsilon\left|\vec{E}\right|^2 dvol$$

$$= 2\frac{1}{4}\int_0^L C_l\left|V(x)\right|^2 dx$$

Both Halves:

$$= \frac{\pi}{8\omega_o}\frac{V_o^{\,2}}{Z_o}$$

Magnetic Energy:

$$W_H = \frac{1}{4}\iiint_{vol} \mu\left|\vec{H}\right|^2 dvol$$

$$= 2\frac{1}{4}\int_0^L L_l\left|I(x)\right|^2 dx$$

$$= \frac{\pi}{8\omega_o}\frac{V_o^{\,2}}{Z_o}$$

Transmission Line Cavity Q

Assume a small resistive loss per unit length $r_L \Omega/\text{m}$ along the walls of the cavity.

Also assume that this loss does not perturb the field distribution of the cavity mode.

$$P_{loss} = 2 \frac{1}{2} \int_0^L r_l \left| I(x) \right|^2 dx$$

Time average

$$= \frac{1}{2} r_l L \frac{V_o^{\,2}}{Z_o}$$

The cavity Q for the <u>fundamental mode</u> of the transmission line cavity is:

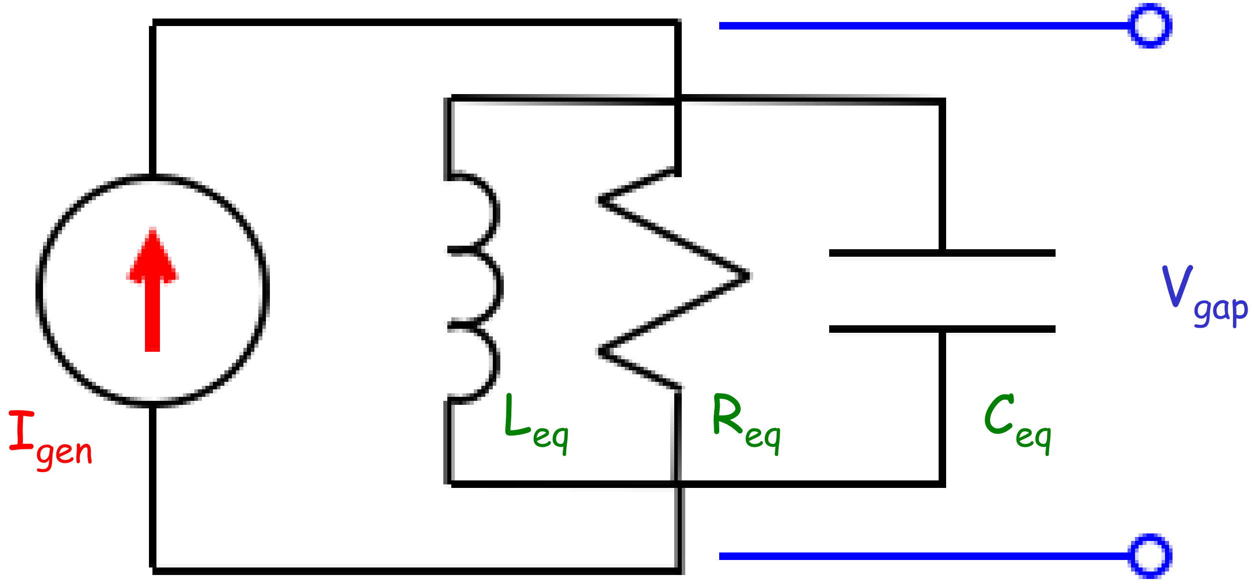
$$Q = \frac{\pi}{2} \frac{Z_o}{r_l L}$$

Less current flowing along walls

Less loss in walls

RLC Model for a Cavity Mode

Around each mode frequency, we can describe the cavity as a simple RLC circuit.



R_{eq} is inversely proportional to the energy lost

L_{eq} is proportional to the magnetic stored energy

C_{eq} is proportional to the electric stored energy

RLC Parameters for a Transmission Line Cavity

For the <u>fundamental mode</u> of the transmission line cavity:

$$P_{loss} = \frac{1}{2}\frac{V_{gap}^{2}}{R_{eq}}$$

$$R_{eq} = 4\frac{Z_o^{2}}{r_l L}$$

$$W_E = \frac{1}{4}C_{eq}V_{gap}^{2}$$

$$C_{eq} = \frac{\pi}{8}\frac{1}{\omega_o Z_o}$$

$$W_H = \frac{1}{4}\frac{V_{gap}^{2}}{\omega_o^{2}L_{eq}}$$

$$L_{eq} = \frac{8}{\pi}\frac{Z_o}{\omega_o}$$

The transfer impedance of the cavity is:

$$Z_c = \frac{V_{gap}}{I_{gen}}$$

$$\frac{1}{Z_c} = \frac{1}{R_{eq}} + \frac{1}{j\omega L_{eq}} + j\omega C_{eq}$$

Cavity Transfer Impedance

Since:

$$\boxed{\frac{1}{\omega_o} = \sqrt{L_{eq} C_{eq}}}$$

Function of geometry only

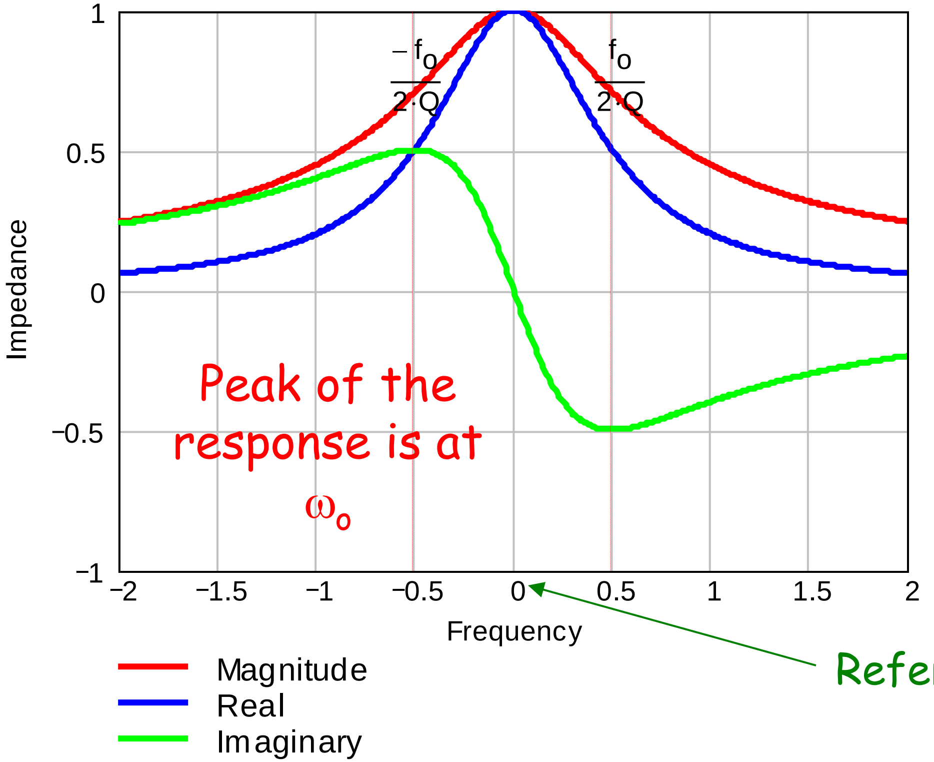
$$\boxed{\frac{R_{eq}}{Q} = \sqrt{\frac{L_{eq}}{C_{eq}}}}$$

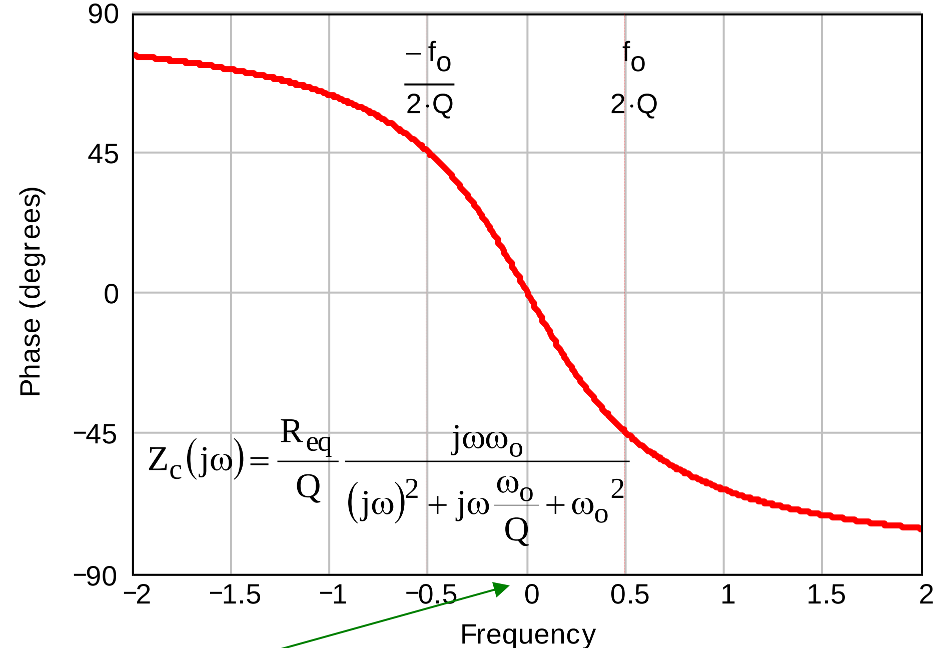
Function of geometry and cavity material

$$\boxed{Q = \omega_o R_{eq} C_{eq}}$$

$$Z_c(j\omega) = \frac{R_{eq}}{Q} \frac{j\omega\omega_o}{(j\omega)^2 + j\omega\frac{\omega_o}{Q} + \omega_o{}^2}$$

Cavity Frequency Response





$$Z_c(j\omega) = \frac{R_{eq}}{Q} \frac{j\omega\omega_o}{(j\omega)^2 + j\omega\frac{\omega_o}{Q} + \omega_o{}^2}$$

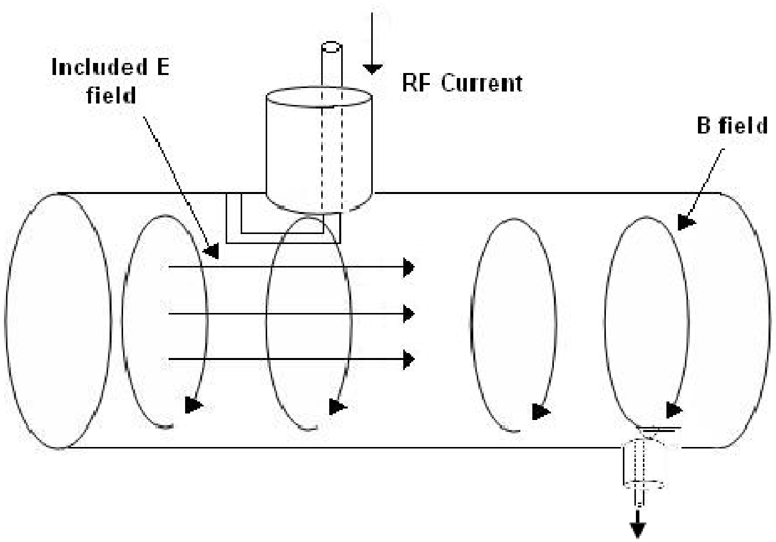
Referenced to ω_o

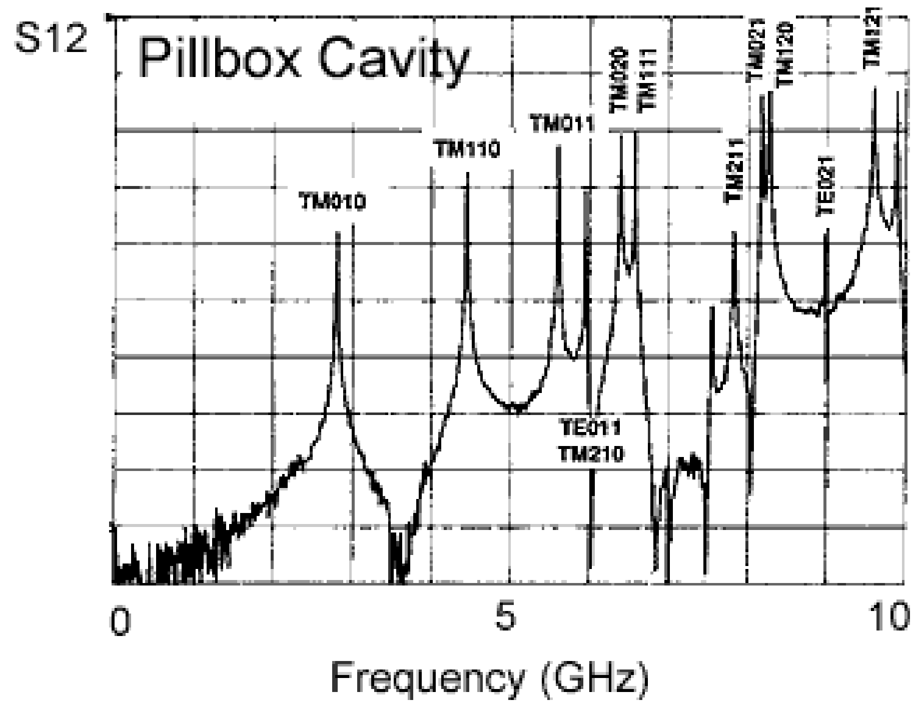
$$\left| Z\left(\omega_o \pm \frac{\omega_o}{2Q}\right)\right|^2 = \frac{1}{2}\left|Z(\omega_o)\right|^2$$

$$\mathrm{Im}\left\{ Z\left(\omega_o \pm \frac{\omega_o}{2Q}\right)\right\} = \mp \mathrm{Re}\left\{ Z\left(\omega_o \pm \frac{\omega_o}{2Q}\right)\right\}$$

$$\arg\left\{ Z\left(\omega_o \pm \frac{\omega_o}{2Q}\right)\right\} = \mp 45°$$

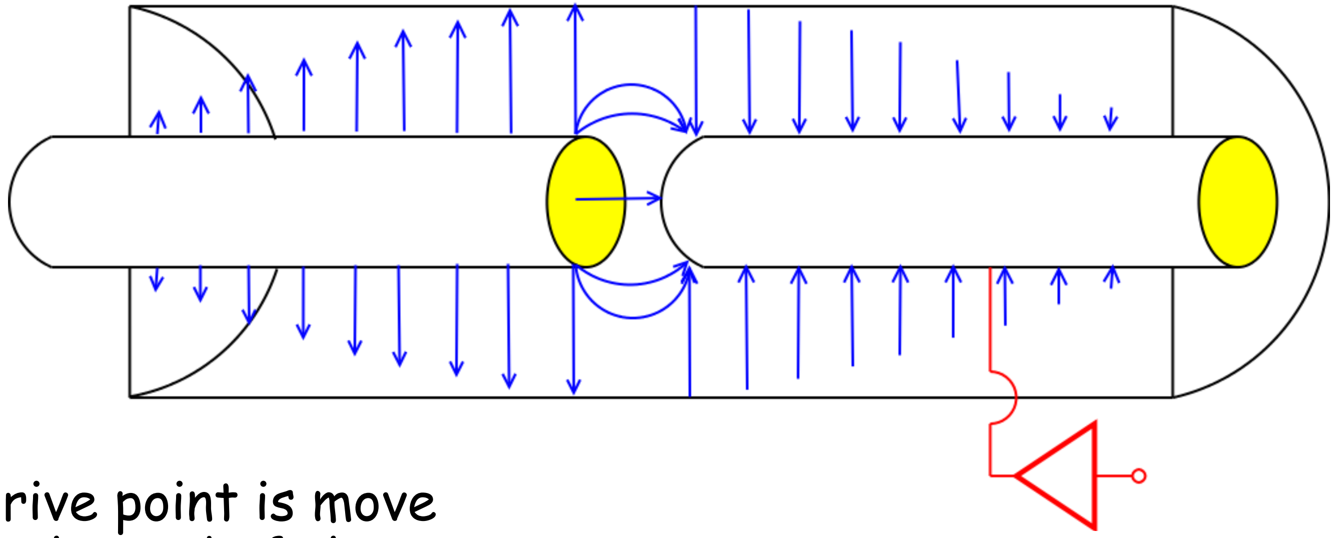
Mode Spectrum Example – Pill Box Cavity



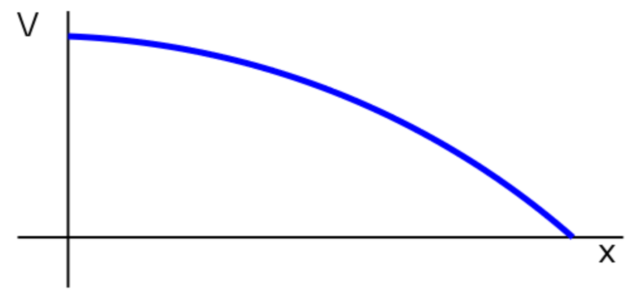


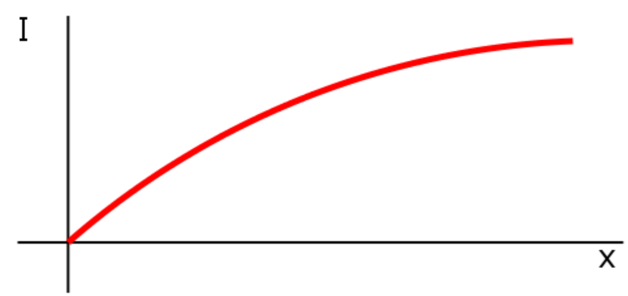
- The RLC model is only around a given mode
- Each mode will a different value of R,L, and C

Cavity Coupling – Offset Coupling

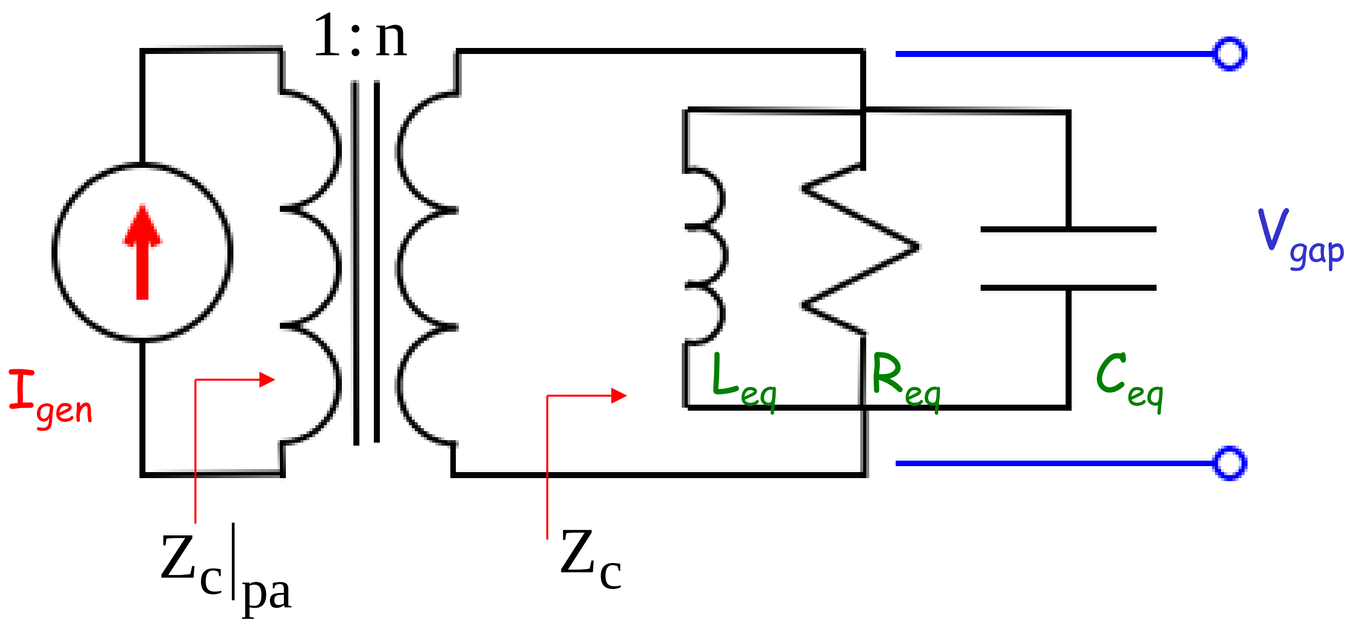


- As the drive point is move closer to the end of the cavity (away from the gap), the amount of current needed to develop a given voltage must increase
- Therefore the input impedance of the cavity as seen by the power amplifier decreases as the drive point is moved away from the gap





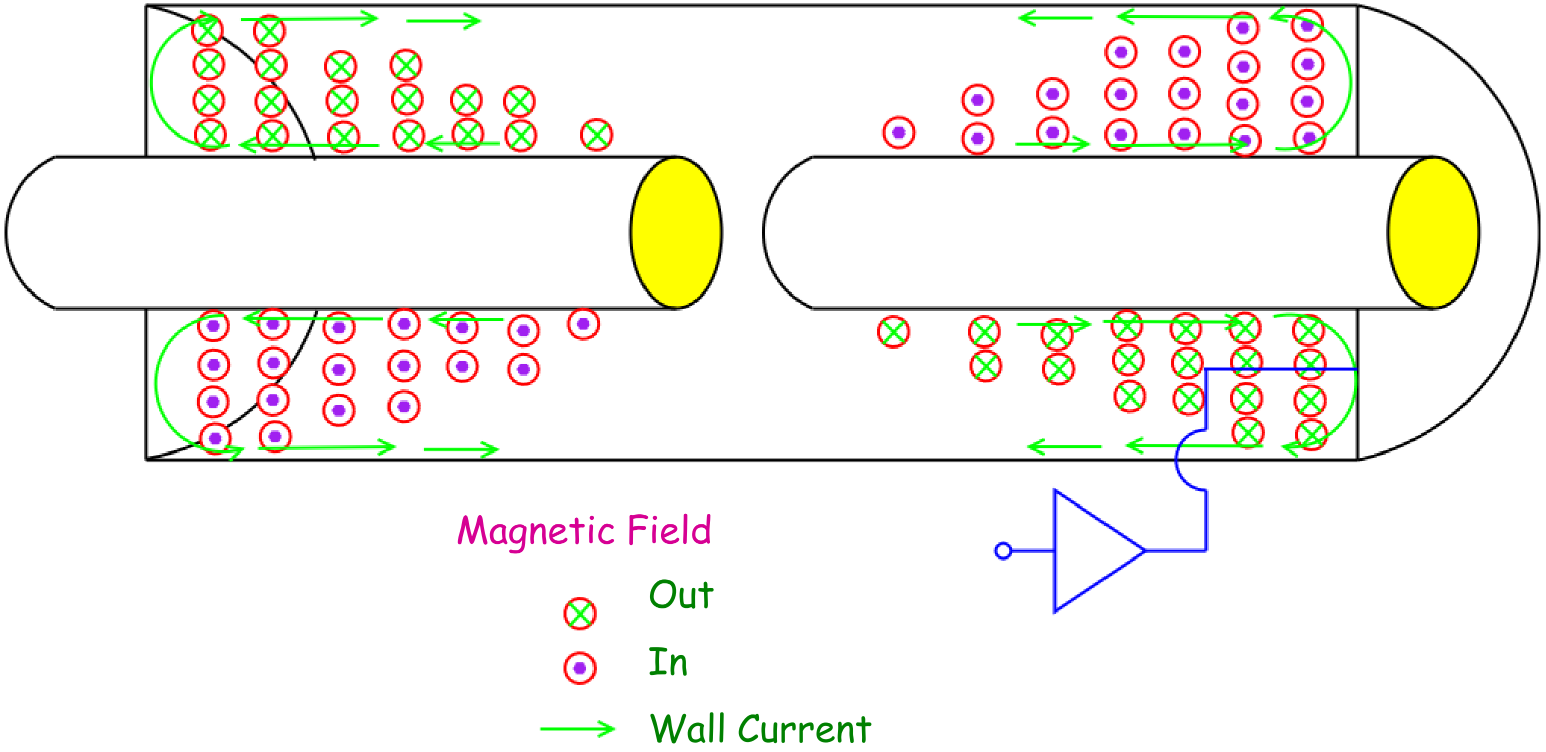
Cavity Coupling



$$Z_c|_{pa} = \frac{1}{n^2} Z_c$$

- We can model moving the drive point as a transformer
- Moving the drive point away from the gap increases the transformer turn ratio (n)

Inductive Coupling



- For inductive coupling, the PA does not have to be directly attached to the beam tube.
- The magnetic flux thru the coupling loop couples to the magnetic flux of the cavity mode
- The transformer ratio n = Total Flux / Coupler Flux

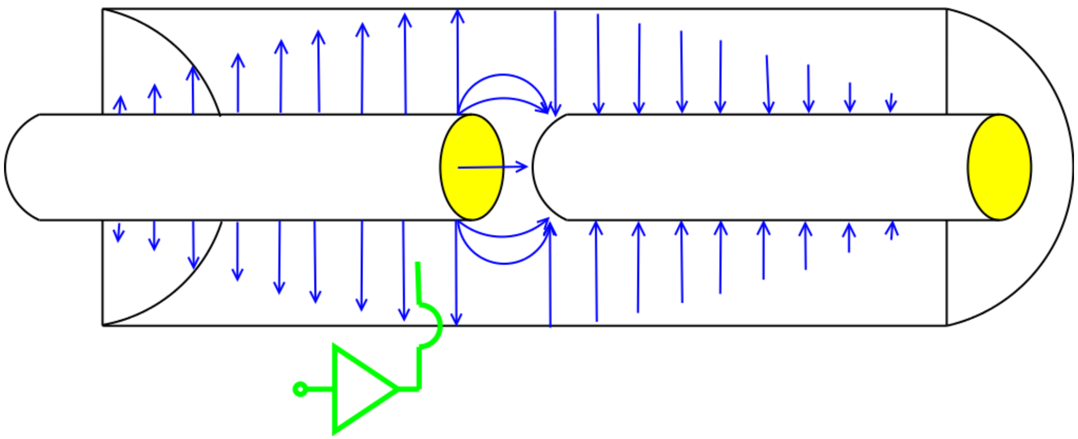
Capacitive Coupling

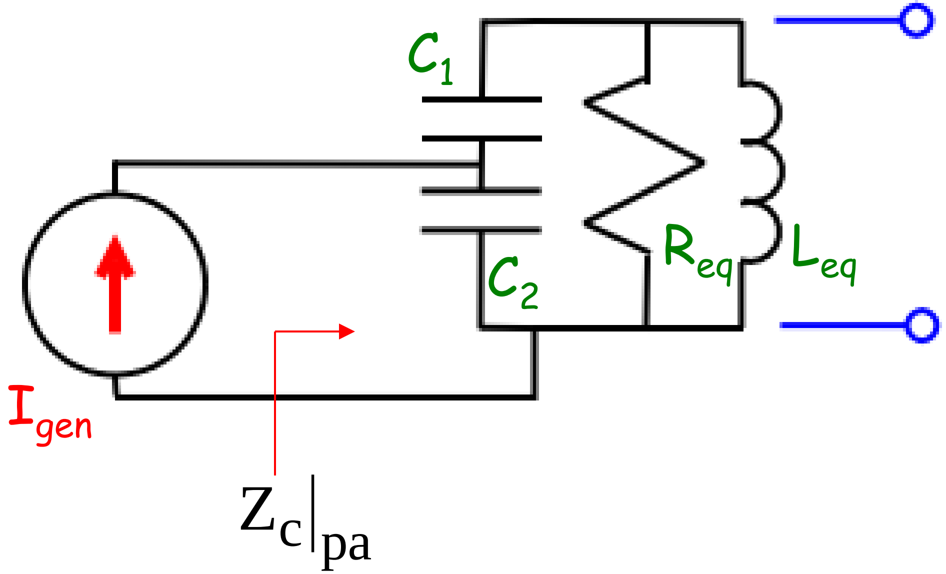
If the drive point does not physically touch the cavity gap, then the coupling can be described by breaking the equivalent cavity capacitance into two parts.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

As the probe is pulled away from the gap, C_2 increases and the impedance of the cavity as seen by the power amp decreases

$$Z_c\big|_{pa} \approx \frac{1}{C_2 / C_{eq}} Z_c$$

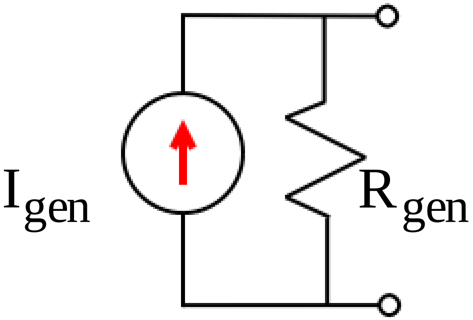




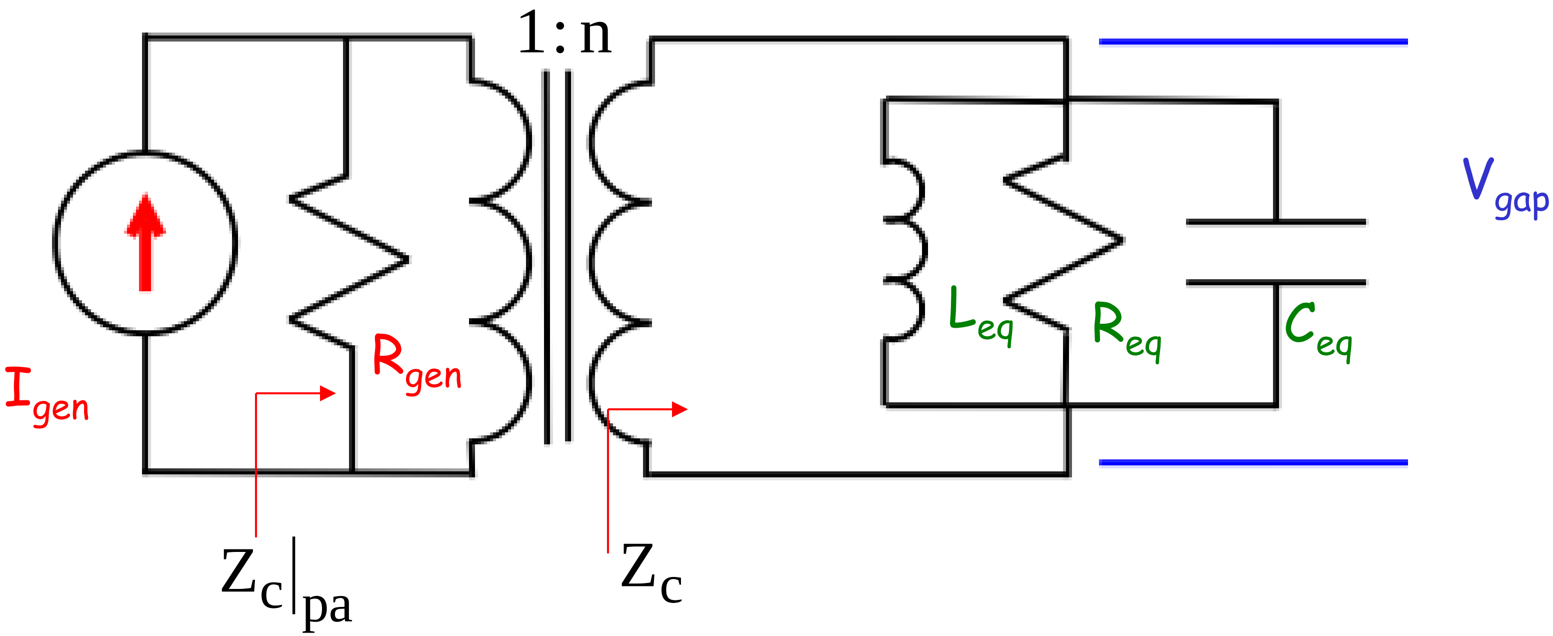
Power Amplifier Internal Resistance

- So far we have been ignoring the internal resistance of the power amplifier.
  - This is a good approximation for tetrode power amplifiers that are used at Fermilab in the Booster and Main Injector
  - This is a bad approximation for klystrons protected with isolators
- Every power amplifier has some internal resistance

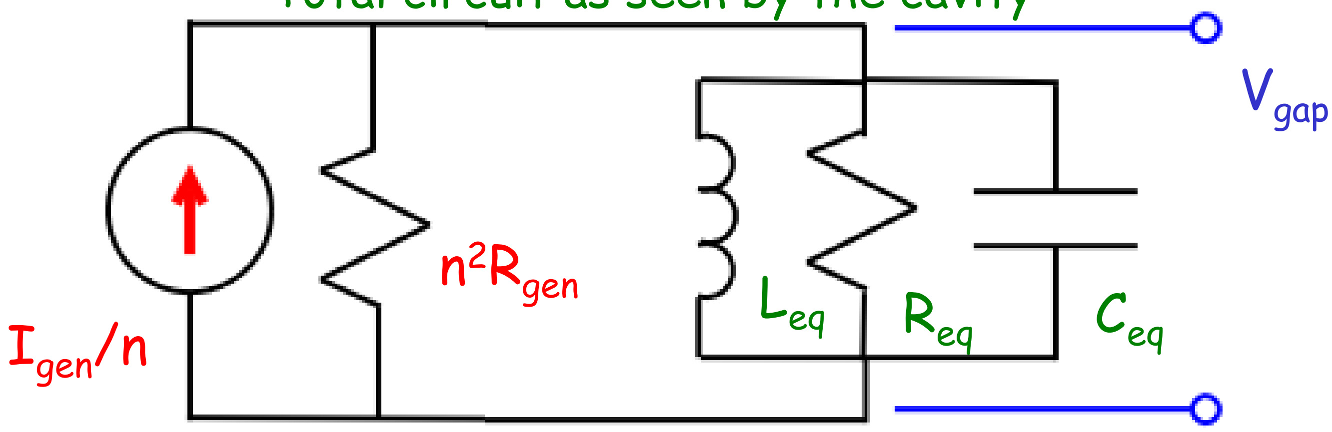
$$R_{gen} = \left.\frac{\Delta V_{gen}}{\Delta I_{gen}}\right|_{I_L=0}$$



Total Cavity Circuit



Total circuit as seen by the cavity



Loaded Q

- The generator resistance is in parallel with the cavity resistance.
- The total resistance is now lowered.

$$\frac{1}{R_L} = \frac{1}{R_{eq}} + \frac{1}{n^2 R_{gen}}$$

- The power amplifier internal resistance makes the total Q of the circuit smaller (d'Q)

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$

$$Q_L = \omega_o R_L C_{eq}$$ **Loaded Q**

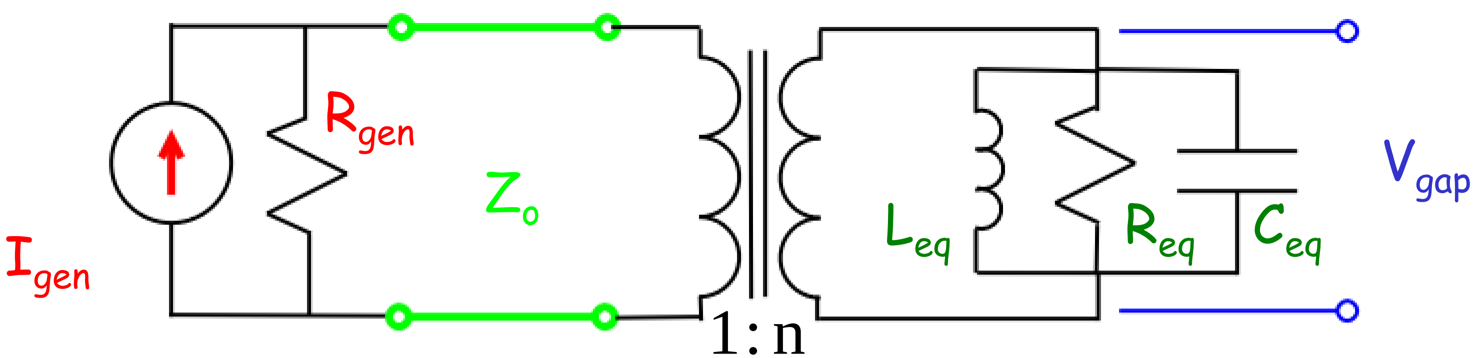
$$Q_o = \omega_o R_{eq} C_{eq}$$ **Unloaded Q**

$$Q_{ext} = \omega_o n^2 R_{gen} C_{eq}$$ **External Q**

Cavity Coupling

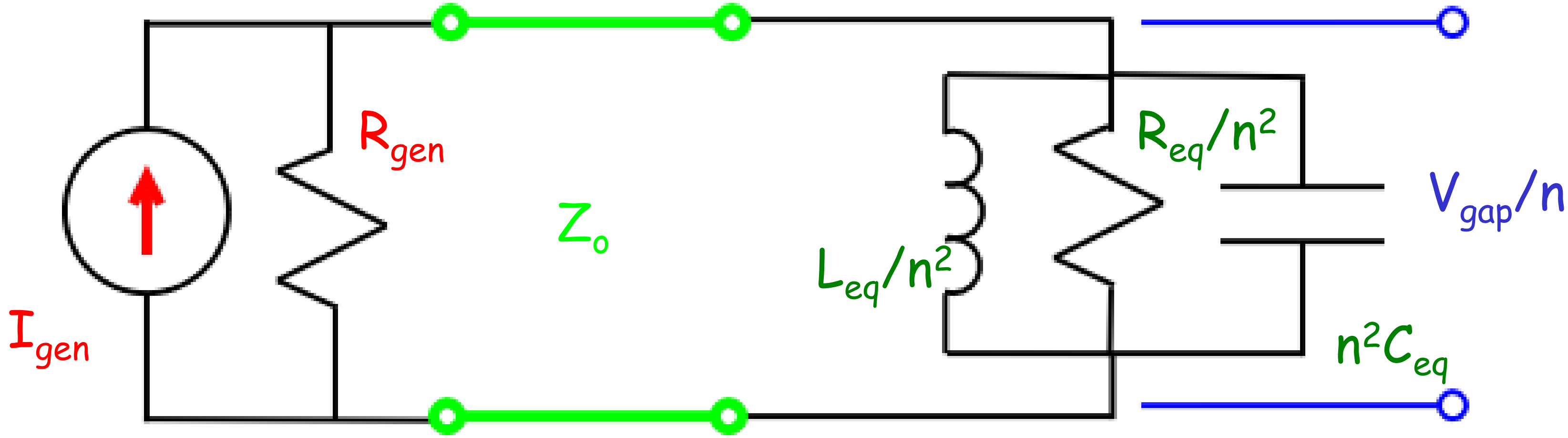
- The cavity is attached to the power amplifier by a transmission line.
  - In the case of power amplifiers mounted directly on the cavity such as the Fermilab Booster or Main Injector, the transmission line is infinitesimally short.
- The internal impedance of the power amplifier is usually matched to the transmission line impedance connecting the power amplifier to the cavity.
  - As in the case of a Klystron protected by an isolator
  - As in the case of an infinitesimally short transmission line

$$R_{gen} = Z_o$$



Cavity Coupling

Look at the cavity impedance from the power amplifier point of view:



Assume that the power amplifier is matched (R_{gen}=Z_o) and define a coupling parameter as the ratio of the real part of the cavity impedance as seen by the power amplifier to the characteristic impedance.

$$r_{cpl} = \frac{R_{eq}/n^2}{Z_o}$$

$r_{cpl} < 1$ **under-coupled**

$r_{cpl} = 1$ **Critically-coupled**

$r_{cpl} > 1$ **over-coupled**

Which Coupling is Best?

- Critically coupled would provide maximum power transfer to the cavity.
- However, some power amplifiers (such as tetrodes) have very high internal resistance compared to the cavity resistance and the systems are often under-coupled.
  - The limit on tetrode power amplifiers is dominated by how much current they can source to the cavity
- Some cavities a have extremely low losses, such as superconducting cavities, and the systems are sometimes over-coupled.
- An intense beam flowing though the cavity can load the cavity which can effect the coupling.

Measuring Cavity Coupling

The frequency response of the cavity at a given mode is:

$$Z_c(j\omega) = \frac{R_{eq}}{Q} \frac{j\omega\omega_o}{(j\omega)^2 + j\omega\frac{\omega_o}{Q} + \omega_o{}^2}$$

which can be re-written as:

$$Z_c(j\omega) = R_{eq}\cos(\phi)e^{j\phi}$$

$$\tan(\phi) = Q\frac{\omega_o{}^2 - \omega^2}{\omega_o\omega}$$

Cavity Coupling

The reflection coefficient as seen by the power amplifier is:

$$\Gamma = \frac{Z_c - n^2 Z_o}{Z_c + n^2 Z_o}$$

$$\Gamma = \frac{r_{cpl}\cos(\phi)e^{j\phi} - 1}{r_{cpl}\cos(\phi)e^{j\phi} + 1}$$

This equation traces out a circle on the reflection (u,v) plane

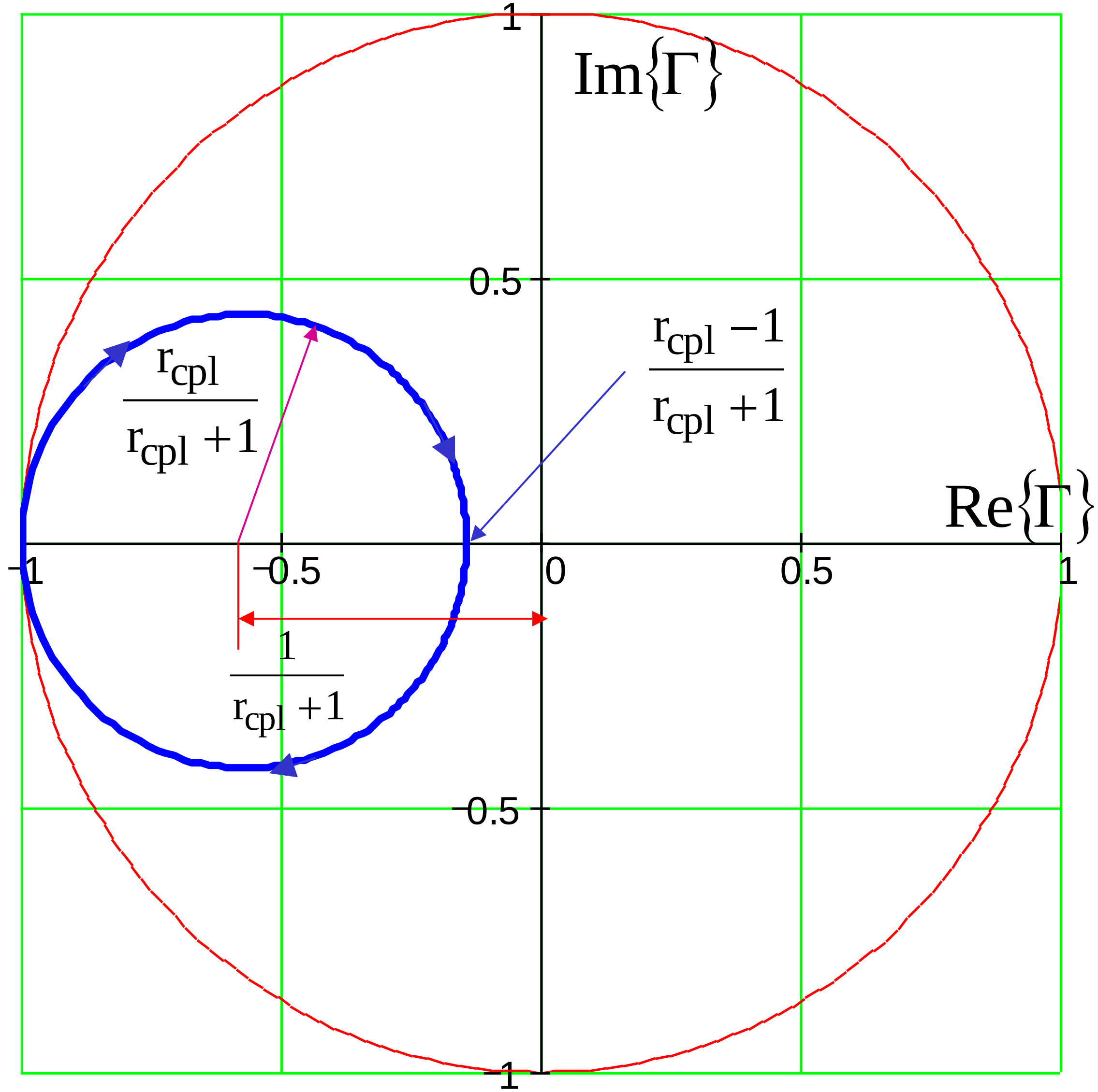
Cavity Coupling

Radius = $\dfrac{r_{cpl}}{r_{cpl}+1}$

Center = $\left(\dfrac{-1}{r_{cpl}+1},0\right)$

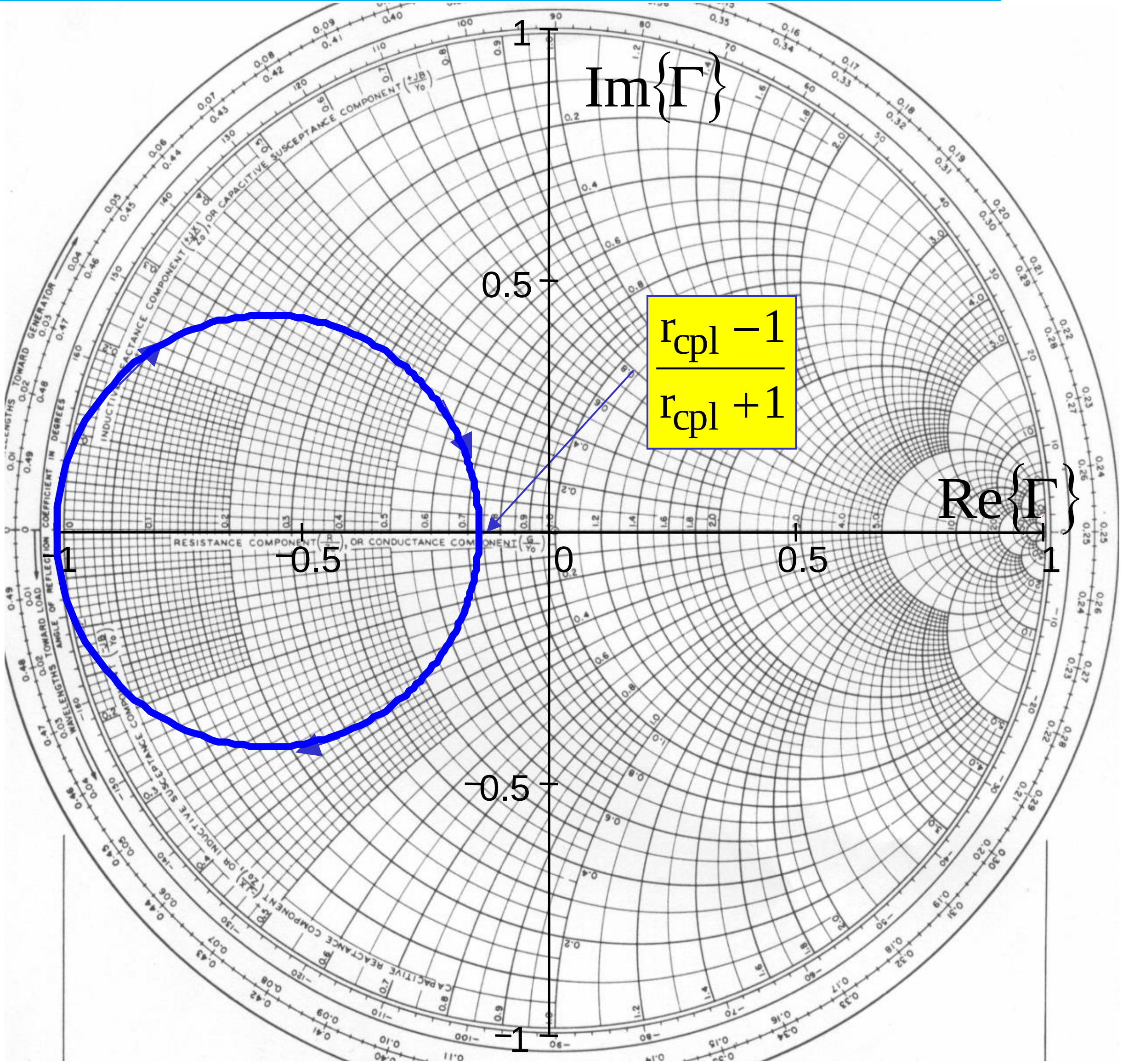
Left edge (ϕ=+/-π/2) = $(-1,0)$

Right edge (ϕ=0) = $\left(\dfrac{r_{cpl}-1}{r_{cpl}+1},0\right)$

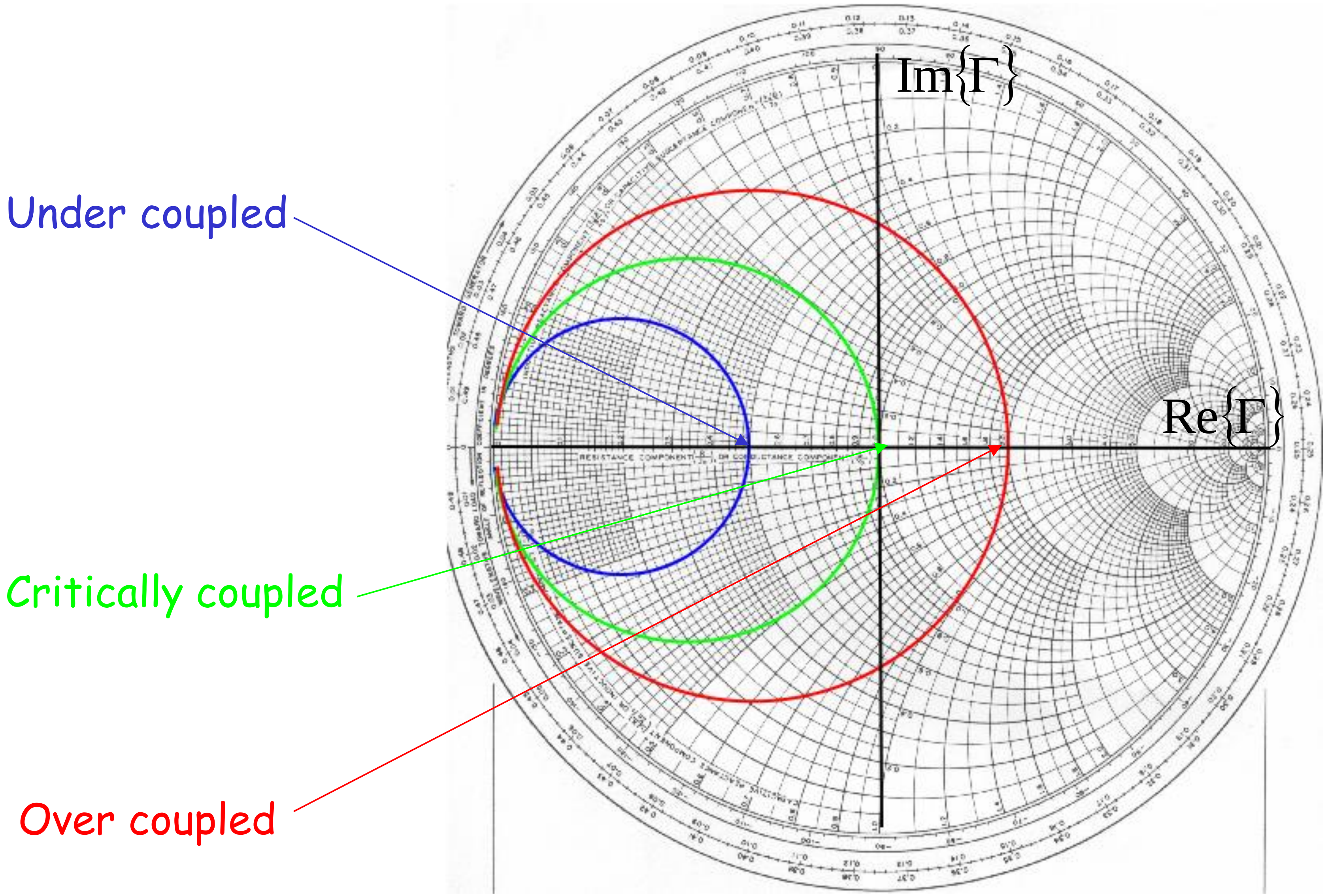


Cavity Coupling

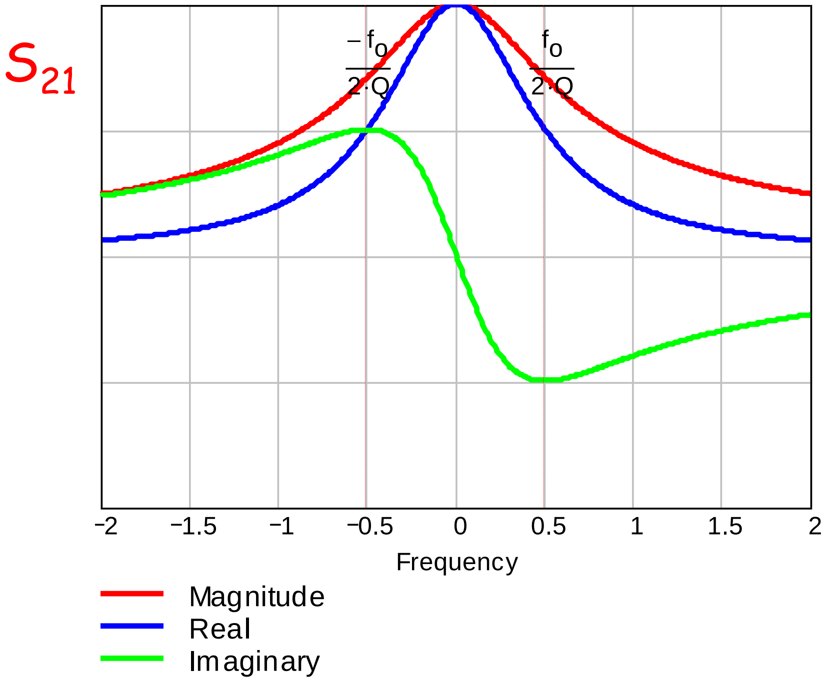
- The cavity coupling can be determined by:
  - measuring the reflection coefficient trajectory of the input coupler
  - Reading the normalized impedance of the extreme right point of the trajectory directly from the Smith Chart

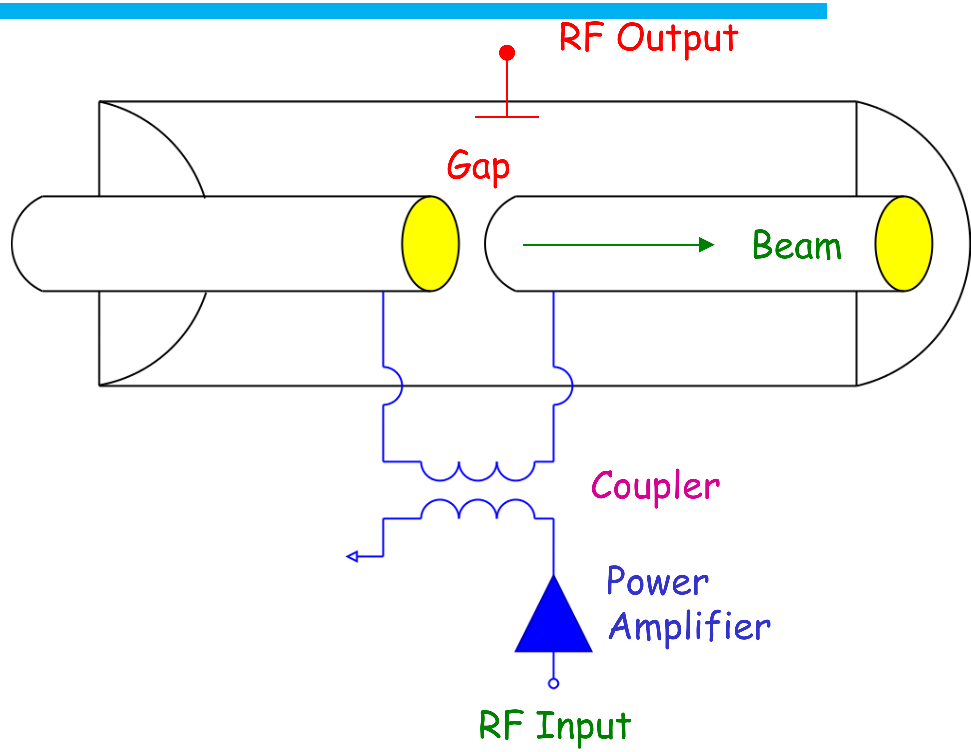


Cavity Coupling



Measuring the **Loaded** Q of the Cavity





- The simplest way to measure a cavity response is to drive the coupler with RF and measure the output RF from a small detector mounted in the cavity.
- Because the coupler "loads" the cavity, this measures the loaded Q of the cavity
  - which depending on the coupling, can be much different than the unloaded Q
  - Also note that changing the coupling in the cavity, can change the cavity response significantly

Measuring the Unloaded Q of a Cavity

- If the coupling is not too extreme, the loaded and unloaded Q of the cavity can be measured from reflection (S_{11}) measurements of the coupler.

At:

$$\omega = \omega_o \mp \frac{\omega_o}{Q_o}$$

Unloaded Q!

$$\phi = \pm \frac{\pi}{4}$$

$$\mathrm{Im}\{Z_c\} = \pm \mathrm{Re}\{Z_c\}$$

For:

$$\mathrm{Im}\{Z_c\} = \pm \mathrm{Re}\{Z_c\}$$

$$u^2 + (v \pm 1)^2 = 2$$

where:

$$\Gamma = u + jv$$

Circles on the Smith Chart

Measuring the <u>Unloaded</u> Q of a Cavity

- Measure frequency (ω_-) when:

  $$\mathrm{Im}\{Z_C\} = \mathrm{Re}\{Z_C\}$$

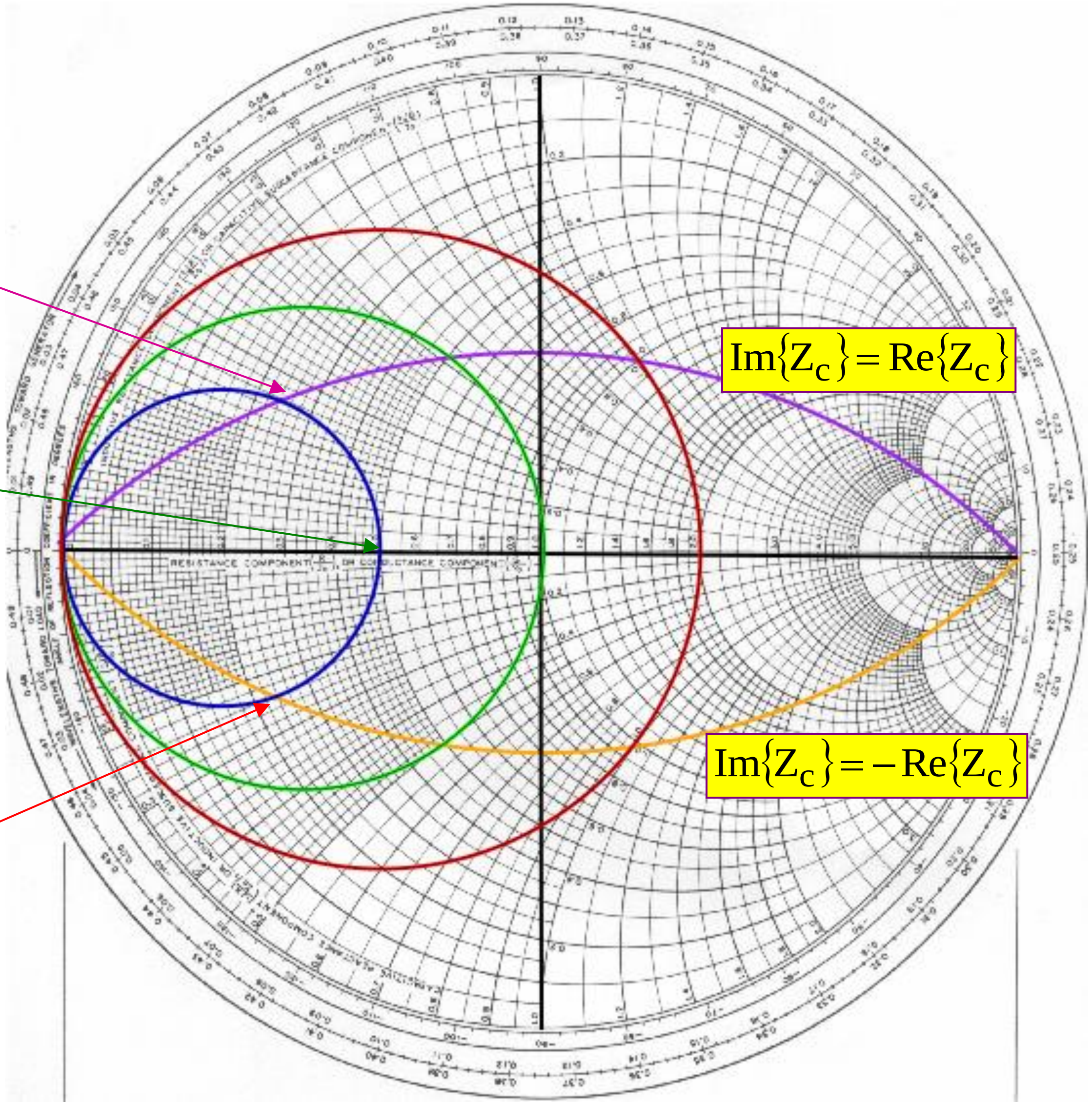
- Measure resonant frequency (ω_o)

- Measure frequency (ω_+) when:

  $$\mathrm{Im}\{Z_C\} = \mathrm{Re}\{Z_C\}$$

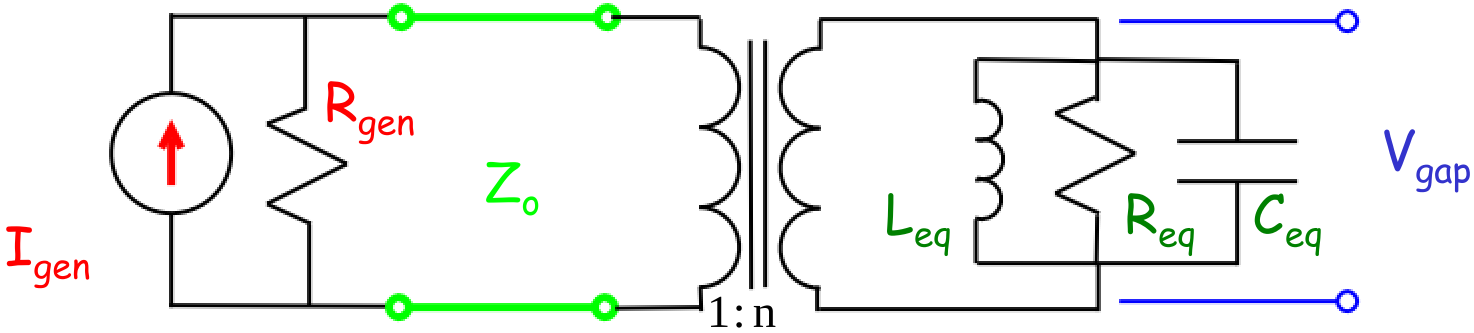
- Compute

  $$Q_o = \frac{\omega_o}{\omega_+ - \omega_-}$$



Measuring the Loaded Q of a Cavity

- Measure the coupling parameter (r_{cpl})
- Measure the unloaded Q (Q_o)



$$R_{eq} = n^2 r_{cpl} Z_o$$

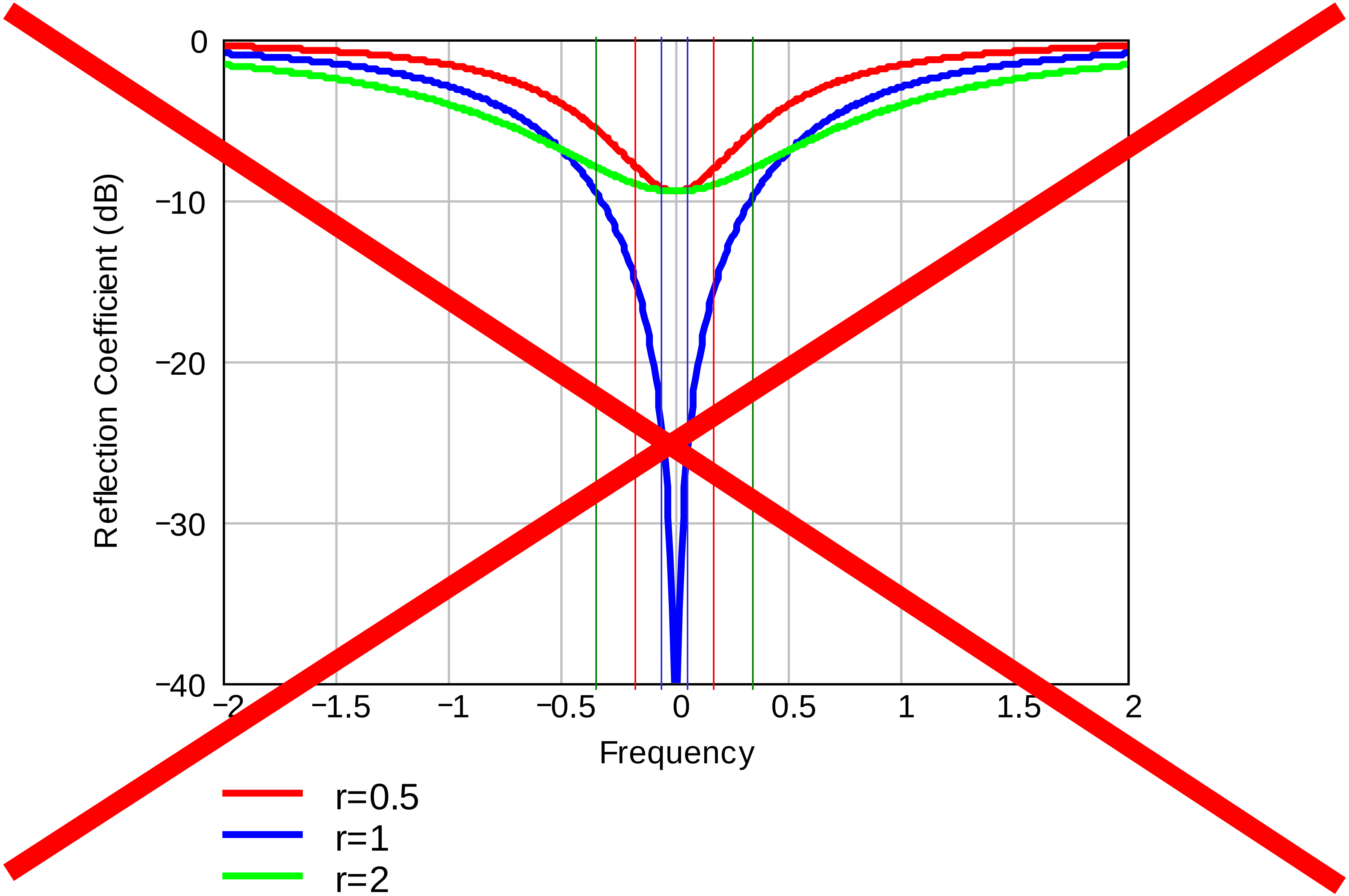
$$Q_o = \omega_o n^2 r_{cpl} Z_o C_{eq}$$

$$Q_{ext} = \omega_o n^2 Z_o C_{eq}$$

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$

$$Q_L = \frac{Q_o}{r_{cpl} + 1}$$

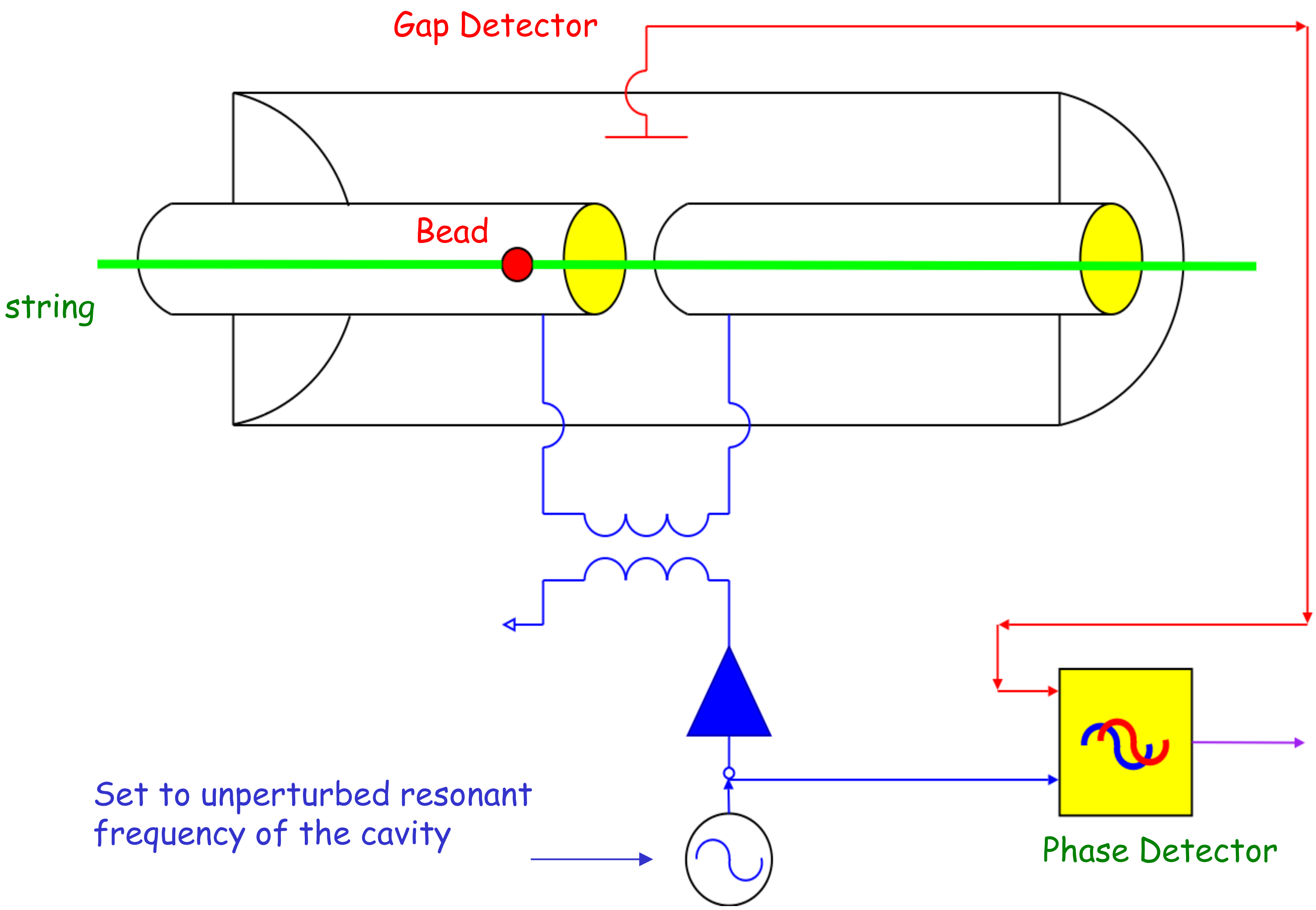
NEVER - EVER!!


0
−10
−20
−30
−40
Reflection Coefficient (dB)
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
Frequency
r=0.5
r=1
r=2

Bead Pulls

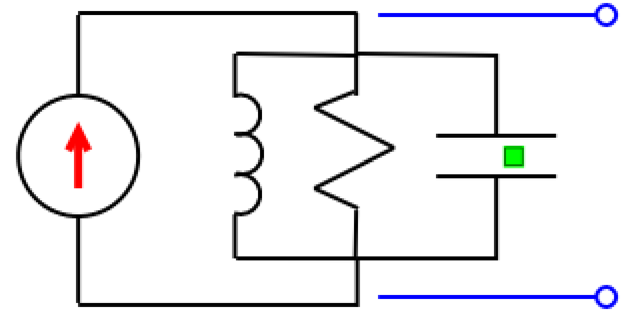
- The Bead Pull is a technique for measuring the fields in the cavity and the equivalent impedance of the cavity as seen by the beam
  - In contrast to measuring the impedance of the cavity as seen by the power amplifier through the coupler

Bead Pull Setup



Bead Pulls

- In the capacitor of the RLC model for the cavity mode consider placing a small dielectric cube
  - Assume that the small cube will not distort the field patterns appreciably
- The stored energy in the capacitor will change



$$W_E = \frac{1}{4} C_{eq} V_{gap}^{\;2} - \frac{1}{4} \varepsilon_o E_c^{\;2} dv + \frac{1}{4} \varepsilon_r \varepsilon_o E_c^{\;2} dv$$

Total original Electric energy

Original Electric energy in the cube

new Electric energy in the cube

Where E_c is the electric field in the cube

dv is the volume of the cube

ε_o is the permittivity of free space

ε_r is the relative permittivity of the cube

$$W_E = \frac{1}{4} \iiint_{vol} \varepsilon \left|\vec{E}\right|^2 dv$$

Bead Pulls

The equivalent capacitance of the capacitor with the dielectric cube is:

$$W_E = \frac{1}{4} C V_{gap}{}^2 = \frac{1}{4}\left(C_{eq} + \Delta C\right) V_{gap}{}^2$$

$$\Delta C = \varepsilon_o \left(\varepsilon_r - 1\right) dv \left(\frac{E_c}{V_{gap}}\right)^2$$

The resonant frequency of the cavity will shift

$$\left(\omega_o + \Delta\omega\right)^2 = \frac{1}{L_{eq}\left(C_{eq} + \Delta C\right)}$$

Bead Pulls

For $\Delta\omega \ll \omega_o$ and $\Delta C \ll C_{eq}$

$$\frac{\Delta\omega}{\omega_o} = \frac{1}{2}\frac{\Delta C}{C_{eq}}$$

$$= \frac{\frac{1}{4}\varepsilon_o(\varepsilon_r - 1)dvE_c^{\,2}}{\frac{1}{2}C_{eq}V_{gap}^{\,2}}$$

$$\frac{\Delta\omega}{\omega_o} = \frac{\Delta W_E}{W_T}$$

Bead Pulls

- Had we used a metallic bead (μ_r>1) or a metal bead:

$$\frac{\Delta\omega}{\omega_o} = \frac{\Delta W_E - \Delta W_H}{W_T}$$

- Also, the shape of the bead will distort the field in the vicinity of the bead so a geometrical form factor must be used.
- For a small <u>dielectric</u> bead of radius a

$$\frac{\Delta\omega}{\omega_o} = -\pi a^3 \varepsilon_o \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right) \frac{{E_b}^2}{W_T}$$

- For a small <u>metal</u> bead with radius a

$$\frac{\Delta\omega}{\omega_o} = -\frac{\pi a^3}{W_T}\left[\varepsilon_o {E_b}^2 + \frac{\mu_o}{2}{H_b}^2\right]$$

A metal bead can be used to measure the E field only if the bead is placed in a region where the magnetic field is zero!

Bead Pulls

- In general, the shift in frequency is proportional to a form factor *F*

$$\frac{\Delta\omega}{\omega_o} = -F\frac{{E_b}^2}{W_T}$$

$$F = \pi a^3 \varepsilon_o \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 2} \right)$$

Dielectric bead

$$F = \pi a^3 \varepsilon_o$$

Metal bead

Bead Pulls

From the definition of cavity Q:

$$Q = \omega_o \frac{W_T}{P_L}$$

$$P_L = \frac{1}{2}\frac{{V_{gap}}^2}{R_{eq}}$$

$$W_T = \frac{1}{2\omega_o}\frac{{V_{gap}}^2}{R_{eq}/Q}$$

$$\left(\frac{E_b(x,y,z)}{V_{gap}}\right)^2 = \frac{1}{F}\frac{1}{2\omega_o}\frac{1}{R_{eq}/Q}\frac{\Delta\omega(x,y,z)}{\omega_o}$$

Bead Pulls

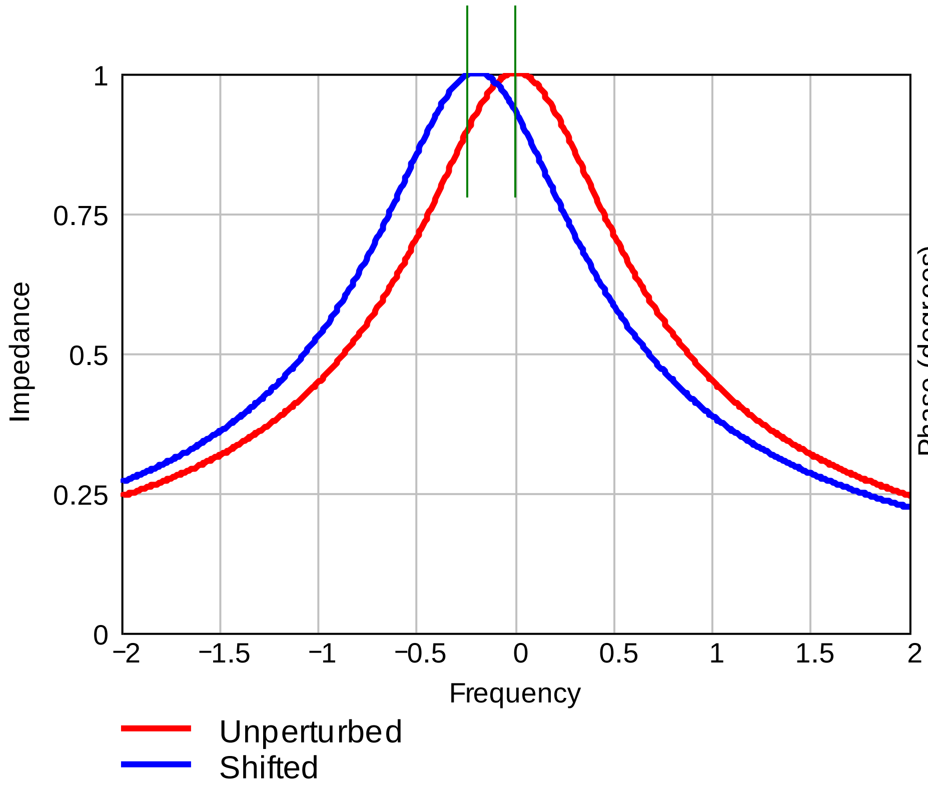
Since:

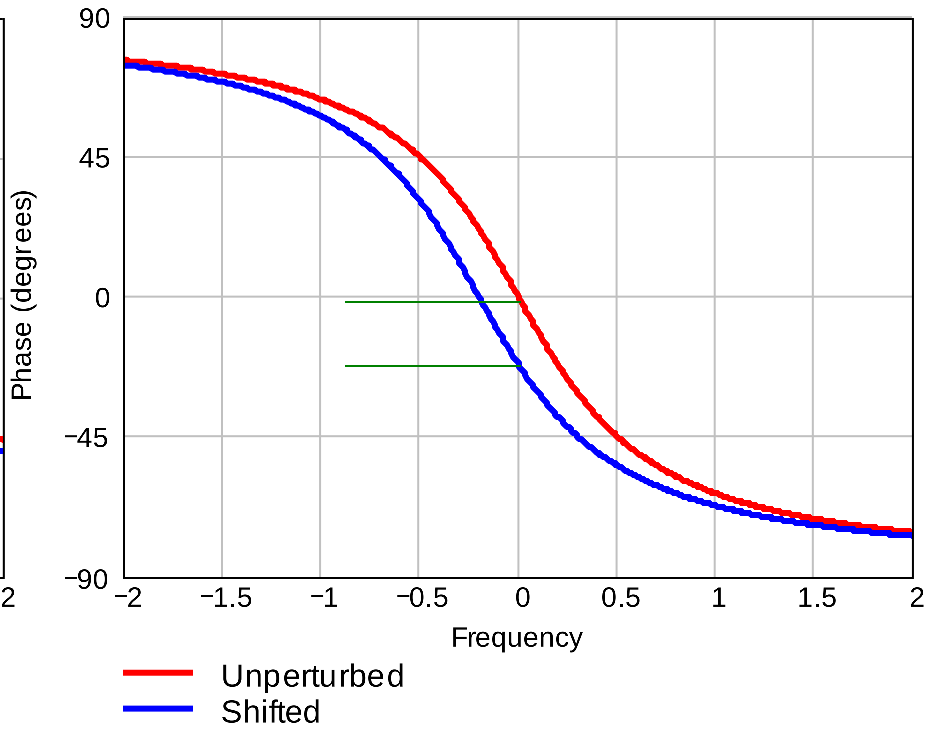
$$\int_{gap} E\left(x_{gap}, y_{gap}, z\right) dz = V_{gap}$$

$$\frac{R_{eq}}{Q} = \frac{1}{F}\frac{1}{2\omega_o}\left[\int_{gap} \sqrt{\frac{\Delta\omega\left(x_{gap}, y_{gap}, z\right)}{\omega_o}}\, dz\right]^2$$

Bead Pulls

- For small perturbations, shifts in the peak of the cavity response is hard to measure.
- Shifts in the phase at the unperturbed resonant frequency are much easier to measure.





Bead Pulls

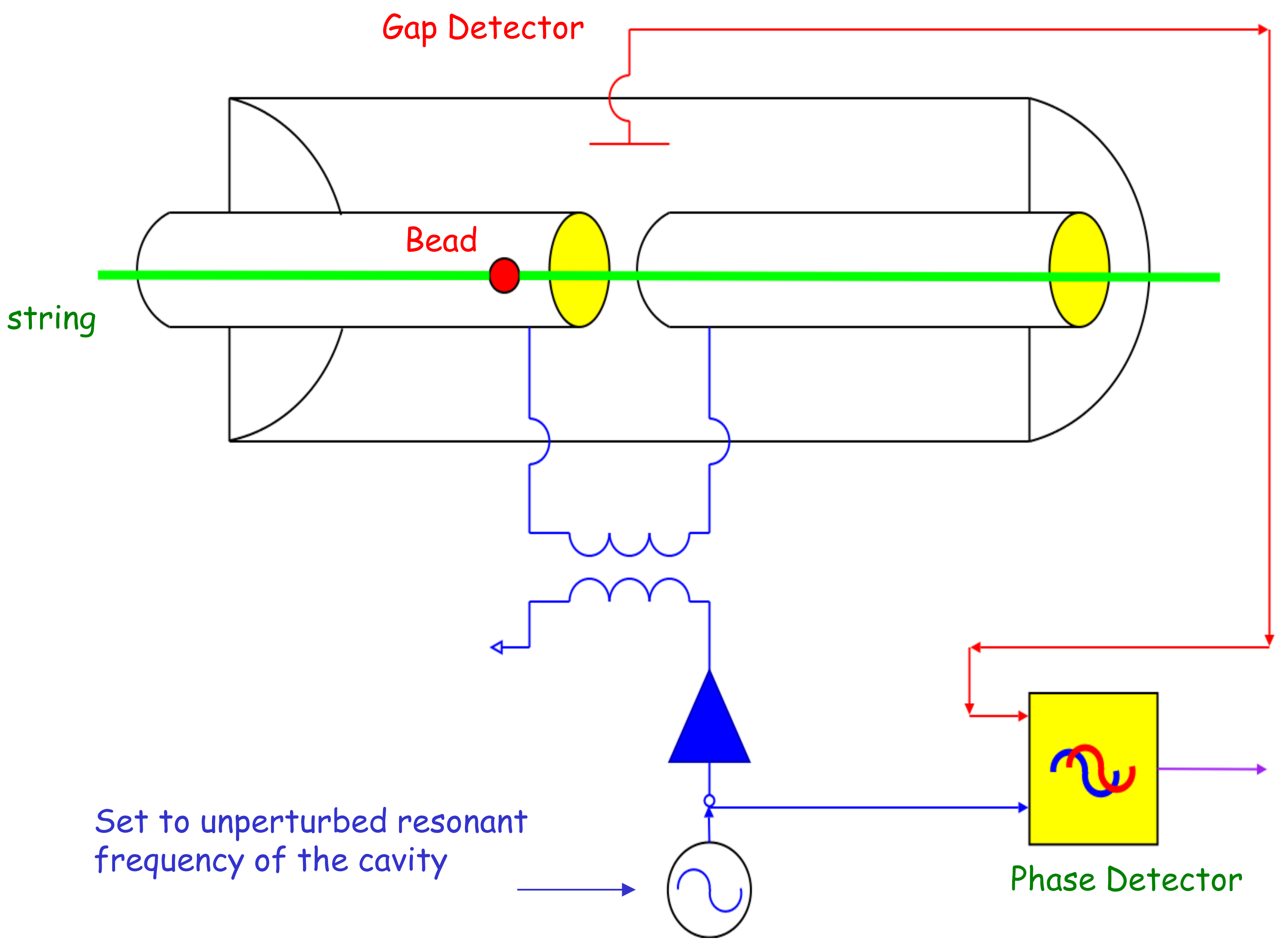
Since:

$$\tan(\phi) = Q\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right)$$

$$\approx 2Q\frac{\Delta\omega}{\omega_o}$$

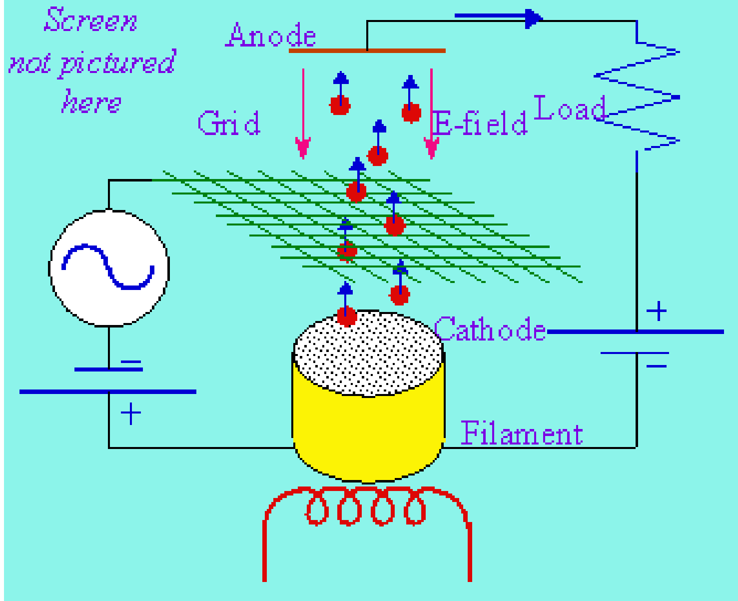
$$R_{eq} = \frac{1}{F}\frac{1}{2\omega_o}\left[\int_{gap}\sqrt{\frac{1}{2}\tan\left(\phi\left(x_{gap}, y_{gap}, z\right)\right)}dz\right]^2$$

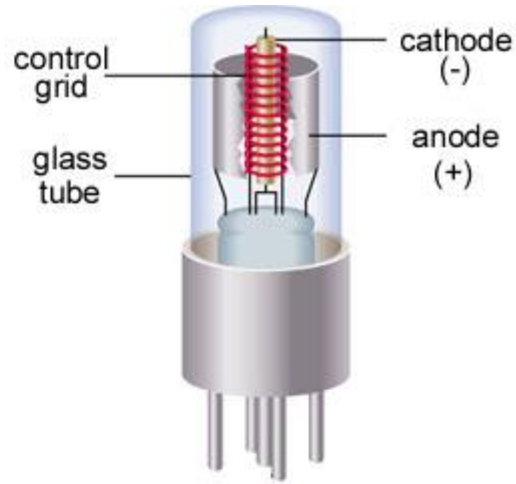
Bead Pulls


Gap Detector
Bead
string
Set to unperturbed resonant
frequency of the cavity
Phase Detector

Power Amplifiers - Triode

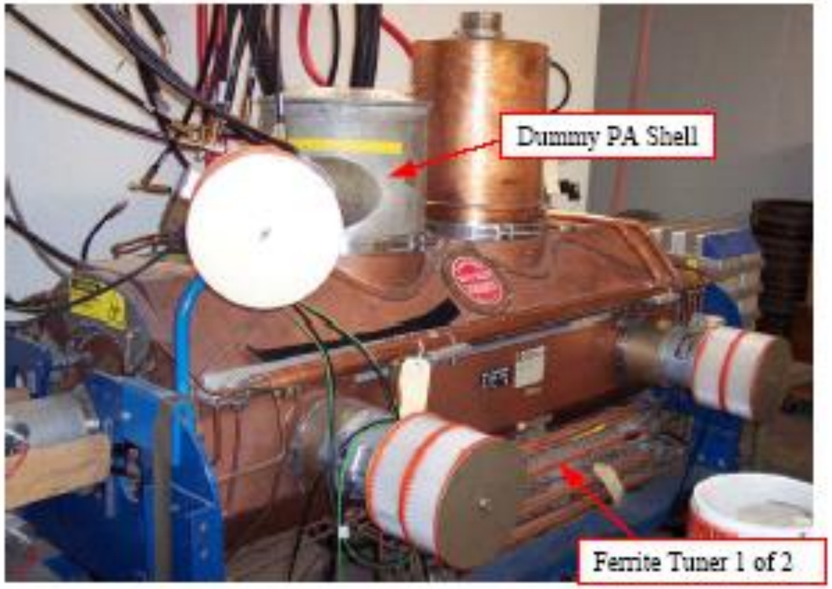
- The triode is in itself a miniature electron accelerator
- The filament boils electrons off the cathode
- The electrons are accelerated by the DC power supply to the anode
- The voltage on the grid controls how many electrons make it to the anode
- The number of electrons flowing into the anode determines the current into the load.
- The triode can be thought of a voltage controlled current source
- The maximum frequency is inversely proportional to the transit time of electrons from the cathode to the anode.
  - Tetrodes are typically used at frequencies below 300 MHz

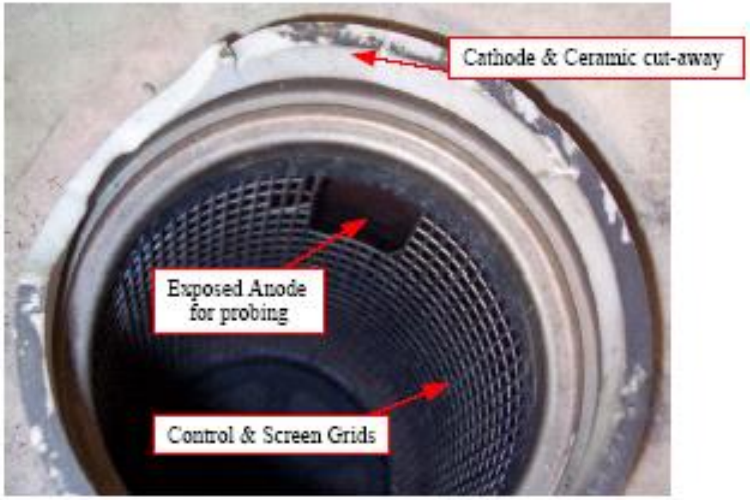




Tetrodes

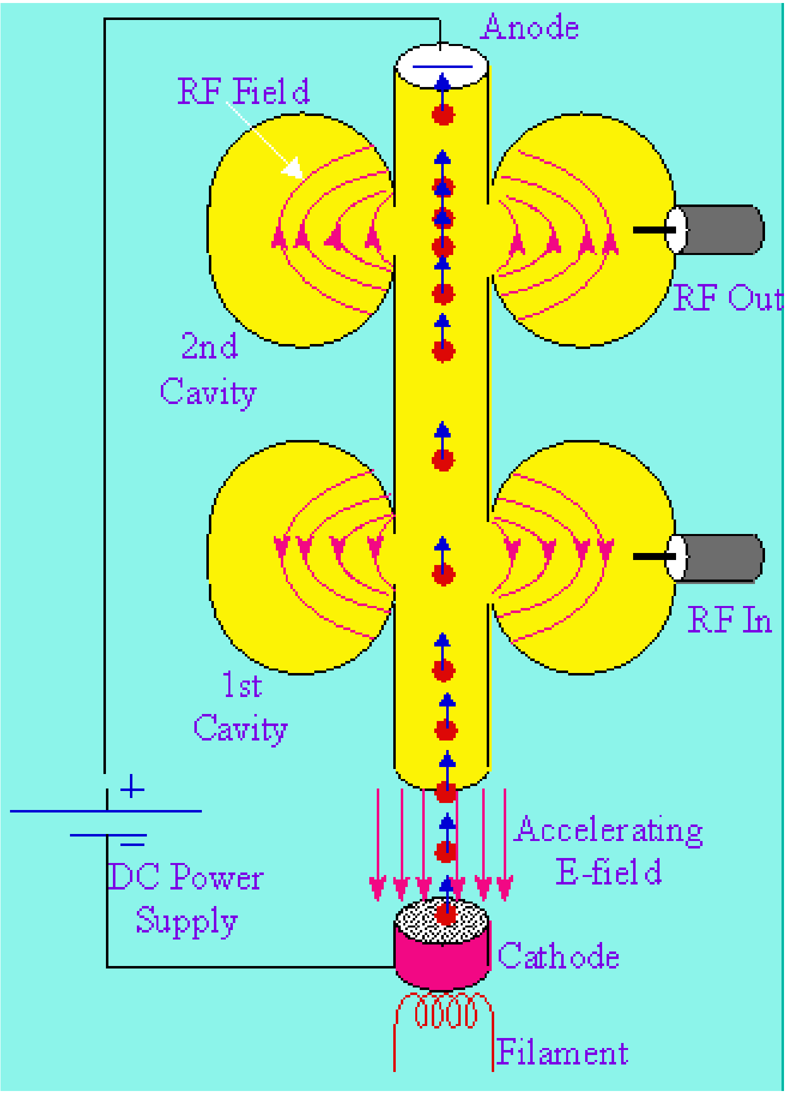
Courtesy of Tim Berenc



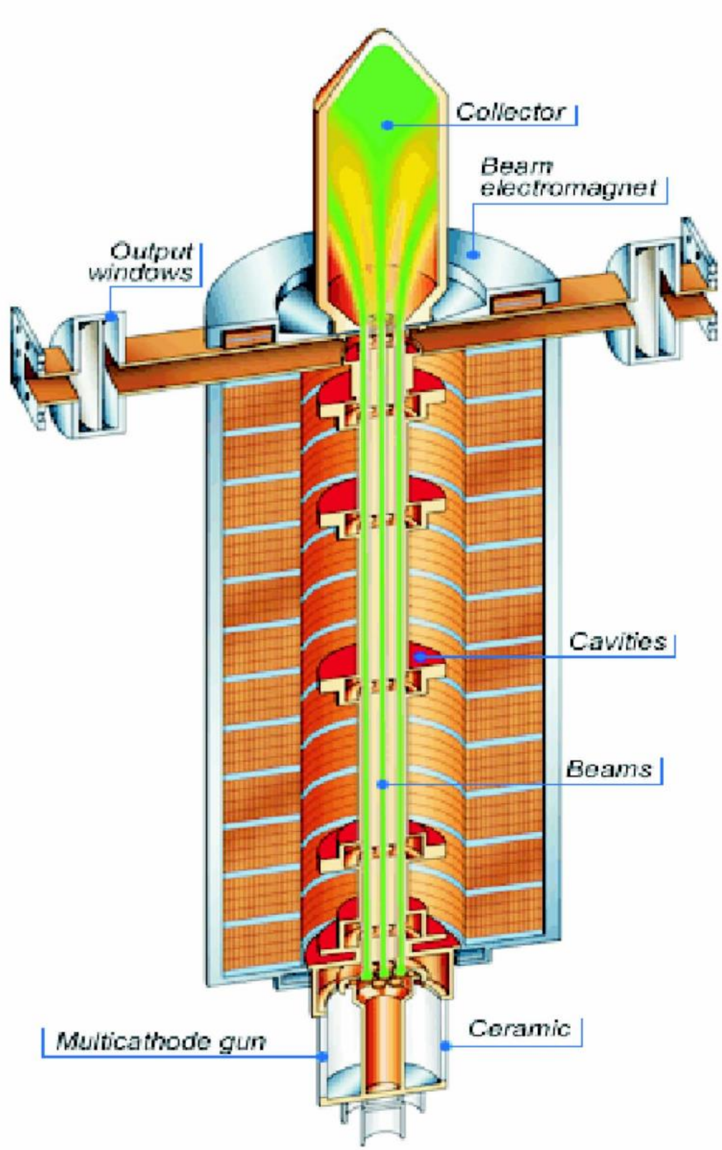


Klystrons

- The filament boils electrons off the cathode
- The velocity (or energy) of the electrons is modulated by the input RF in the first cavity
- The electrons drift to the cathode
- Because of the velocity modulation, some electrons are slowed down, some are sped up.
- If the output cavity is placed at the right place, the electrons will bunch up at the output cavity which will create a high intensity RF field in the output cavity
- Klystrons need a minimum of two cavities but can have more for larger gain.
- A Klystron size is determined by the size of the bunching cavities.
  - Klystrons are used at high frequencies (>500 MHz))



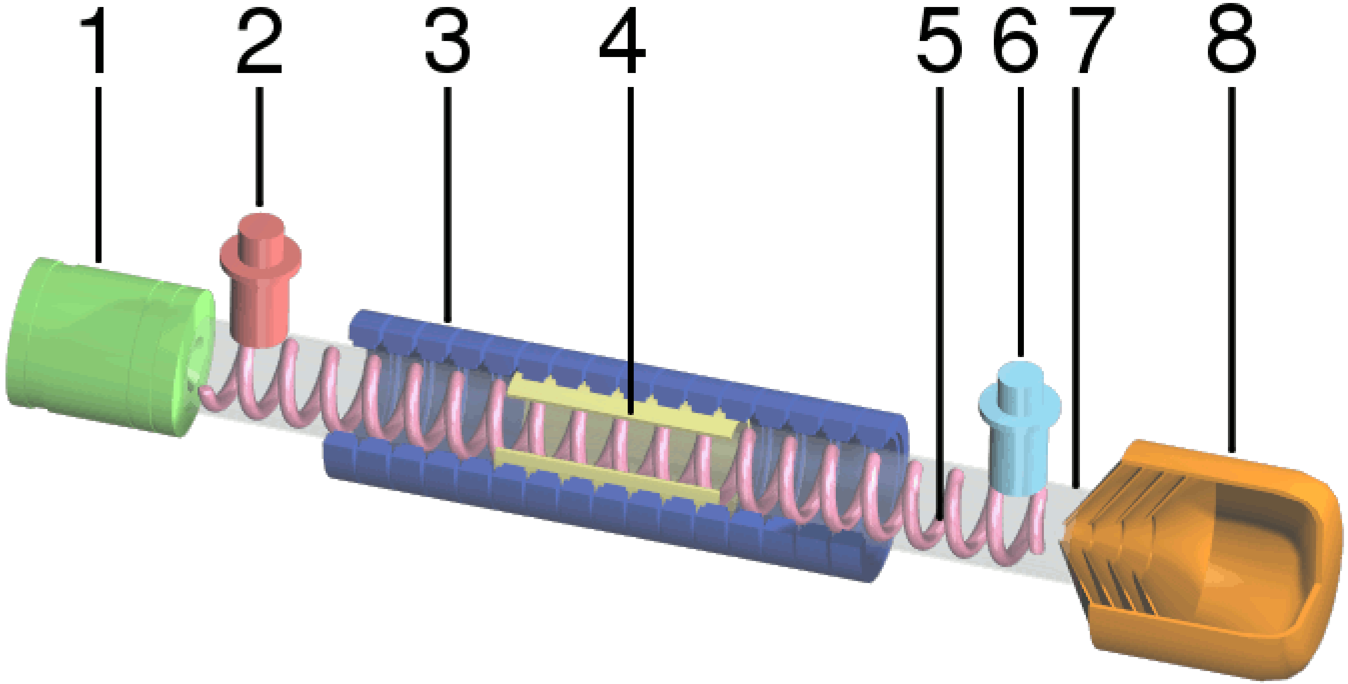
Klystrons



Collector
Beam electromagnet
Output windows
Cavities
Beams
Ceramic
Multicathode gun

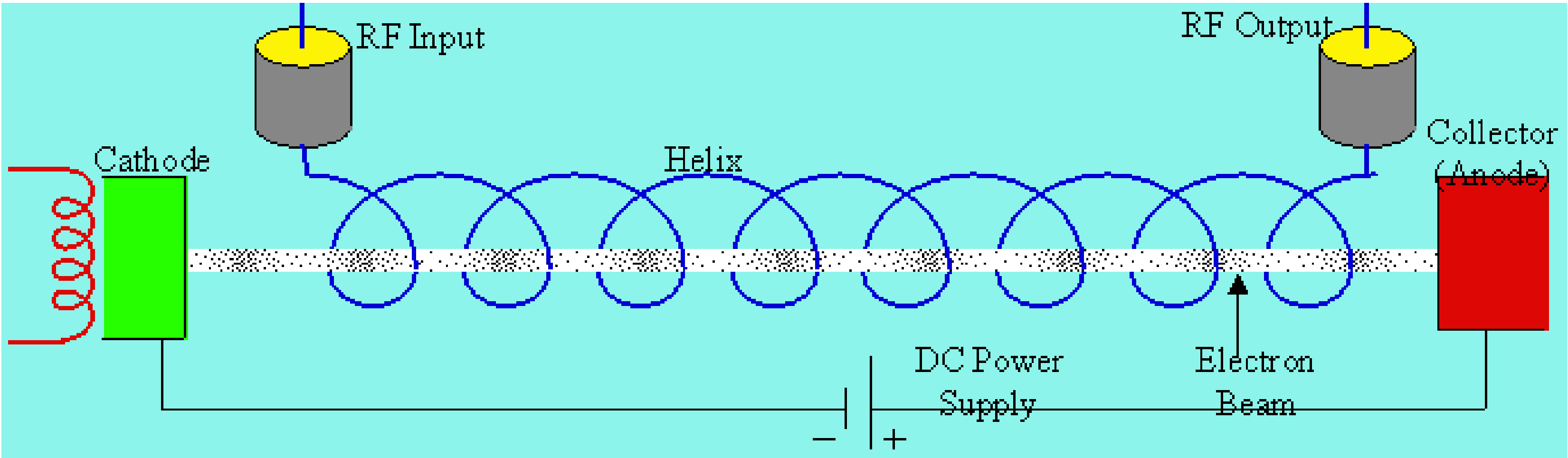
Traveling Wave Tube

Cutaway view of a TWT. (1) Electron gun; (2) RF input; (3) Magnets; (4) Attenuator; (5) Helix coil; (6) RF output; (7) Vacuum tube; (8) Collector.



Traveling Wave Tubes

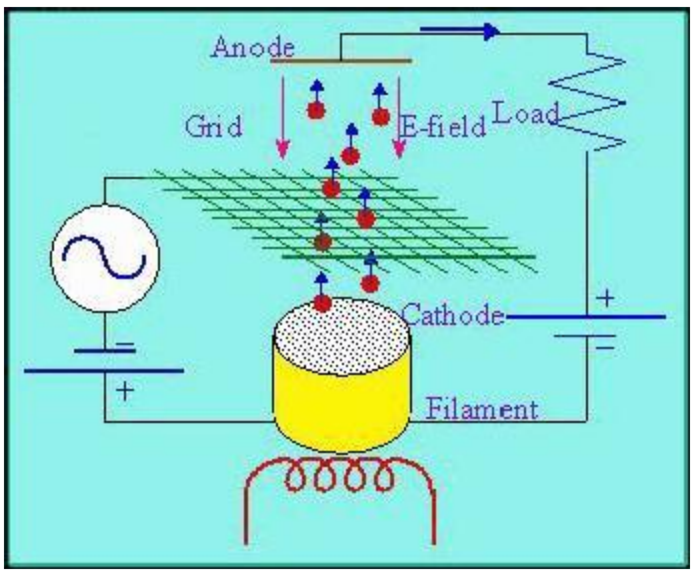
- Traveling wave tubes (TWTs) can have bandwidths as large as an octave ($f_{max} = 2 \times f_{min}$)
- TWTs have a helix which wraps around an electron beam
  - The helix is a slow wave electromagnetic structure.
  - The phase velocity of the slow wave matches the velocity of the electron beam
- At the input, the RF modulates the electron beam.
- The beam in turn strengthens the RF
- Since the velocities are matched, this process happens all along the TWT resulting in a large amplification at the output (40dB = 10000 x)

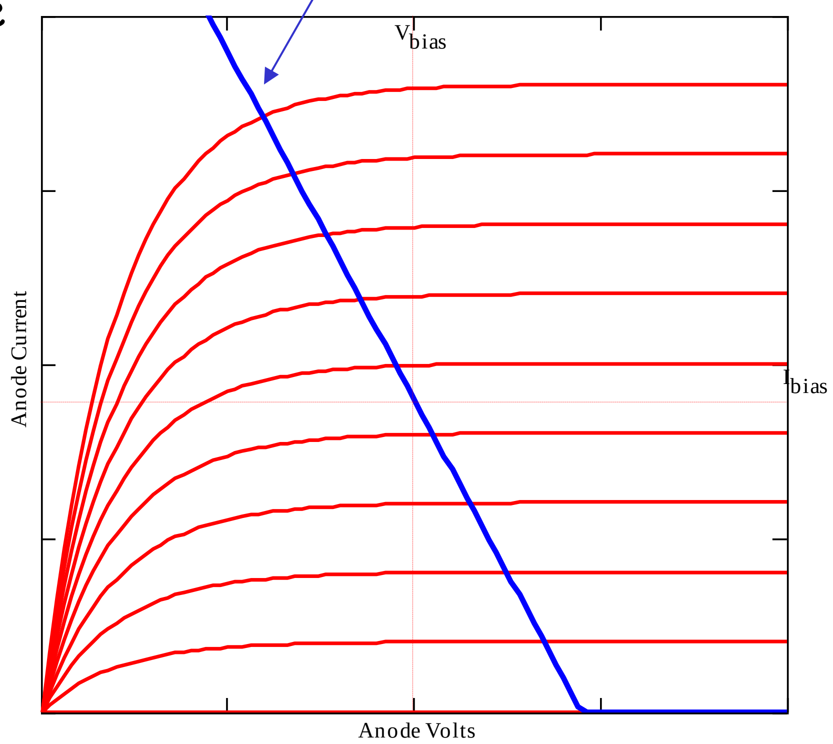


Power Amplifier Bias

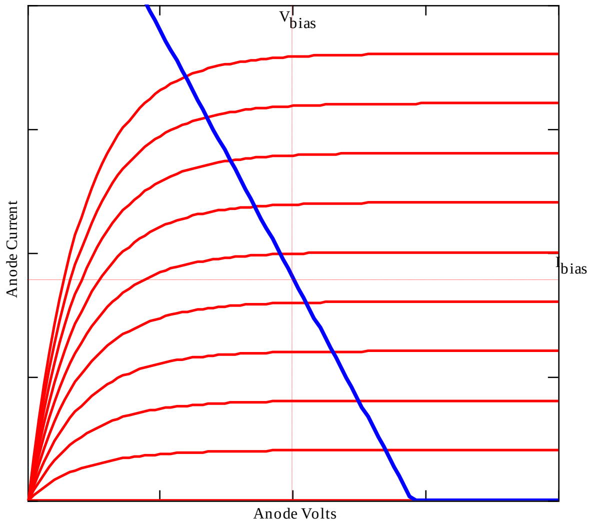
- The power amplifier converts DC energy into RF energy.
- With no RF input into the amplifier, the Power amplifier sits at its DC bias.
- The DC bias point is calculated from the intersection of the tube characteristics with the outside load line

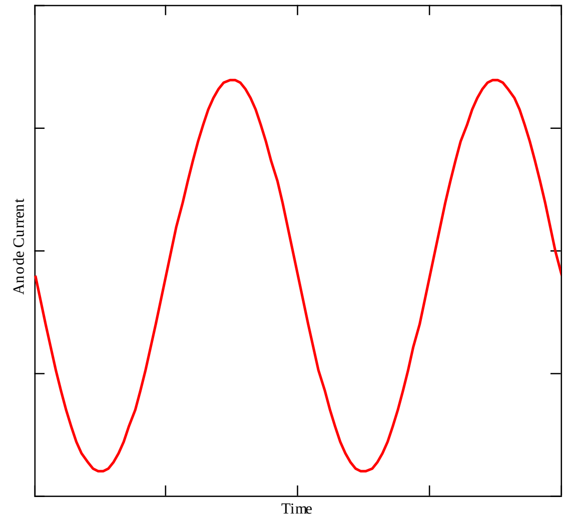
$$I_A = \frac{V_{A_o} - V_A}{R} + I_{A_o}$$

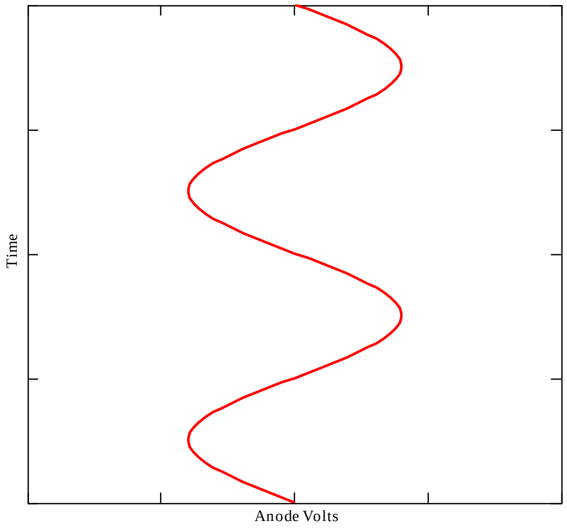




Class A Bias





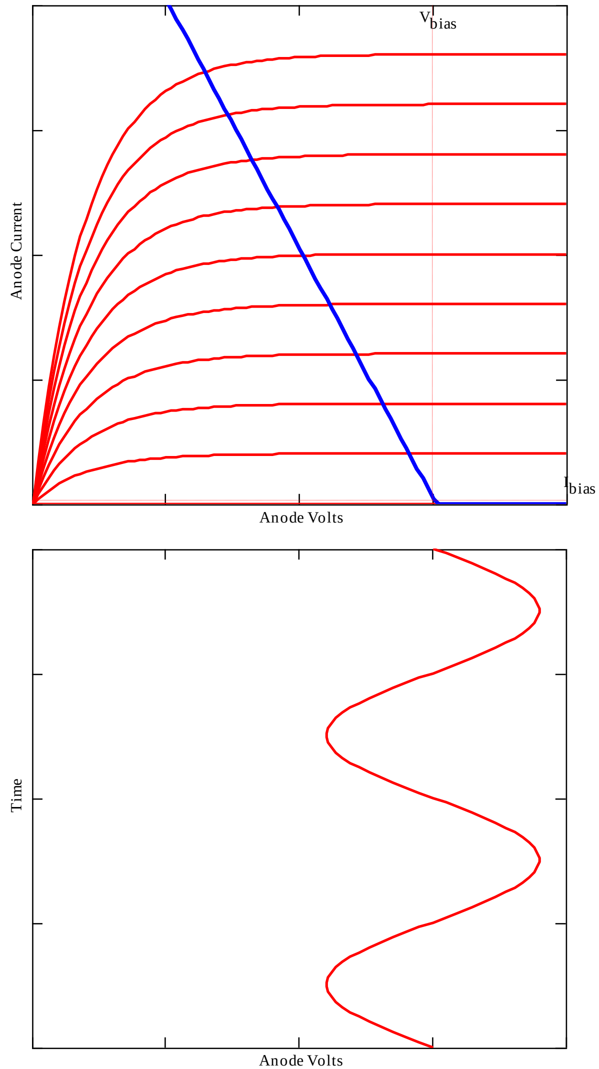


- In Class A, the tube current is on all the time – even when there is no input.
- The tube must dissipate

$$P_{dis} = I_{A_{dc}} V_{A_{dc}} - \frac{1}{2}{}_{A_{ac}} V_{A_{ac}}$$

- The most efficient the power amplifier can be is 50%

Class B Bias



- In Class B, the tube current is on ½ of the time
- The tube dissipates no power when the there is no drive
- The output signal harmonics which must be filtered
- The efficiency is much higher (>70%)