* Radio Frequency & Microwave Physics
Concepts & Techniques

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* 23 years of particle accelerator experience
1) Stochastic cooling systems
2) Beam Feedback systems
3) Beam Instrumention & Control

* 3 years Fermilab Center for Particle Astrophysics
3-D map of univererse $z = .5$ to 2
using Hydrogen 21 cm line.

Course Objectives

Aimed at students working in RF - millimeter wave detectors & high speed electronics.

1) Practical overview of EM theory
2) Guided wave propagation
3) Communication Concepts
4 Devices
5) Antenna

Syllabus

1) Week 1 Electromagnetic Basics
   a) Maxwell's Eqns
   b) Sources & Duality
   c) Uniqueness, Images, Equivalence Induction & Reciprocity
   d) Green's Functions

2) Week 2-3 Guided Waves
   a) Transmission lines
   b) Optimum Power Matching.
      i) Time domain Reflectometer
      ii) Single Stub Tuners
   c) Waveguides
      i) mode sets
      ii) Rectangular & Circular waveguides
      iii) loaded waveguides

3) Week 4 Communication Concepts
   a) Power Spectral Density
   b) Modulation Techniques
   c) Band limited Noise
      i) Noise Temperature
      ii) Noise Figure

4) Week 5-6 Devices

a) S-parameters
b) Passive devices
   Couplers, hybrids, isolators, etc
c) RF cavities
d) Active Devices
   Klystrons, Tetrodes,
   Travelling Wave Tubes

5) Week 7-8 Antennas

a) Patterns, Directivity, Aperture
b) Wire Antennas
c) Phased Arrays
d) Aperture Antennas

References

1) Time Harmonic Electromagnetic Fields by Roger F. Harrington

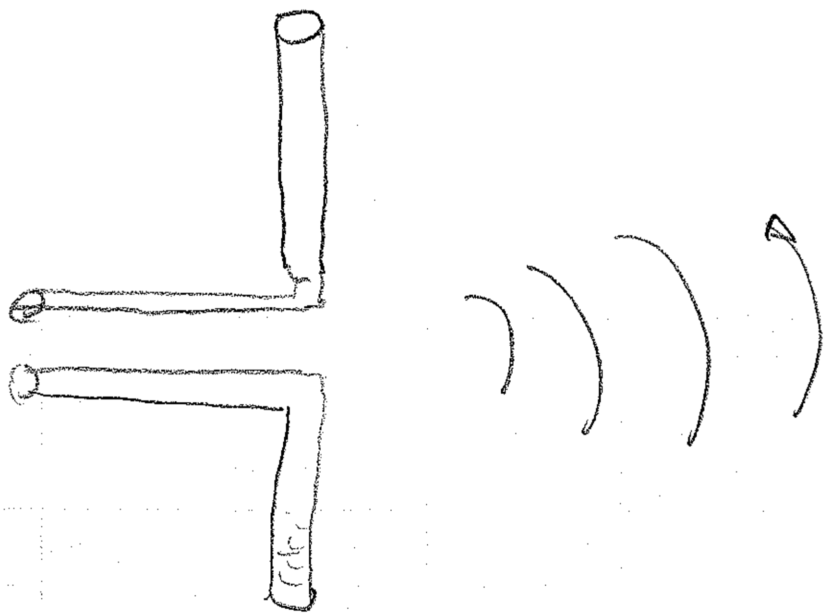
2) Foundations of Microwave Engineering by R. E. Collin

3) Field Theory of Guided Waves by R.E. Collin

4.) Lines, Waves, & Antennas by Brown, Sharpe, Hughes, & Post

5) Antenna Theory & Design by Stutzman & Thièle.

How Does an an antenna radiate?



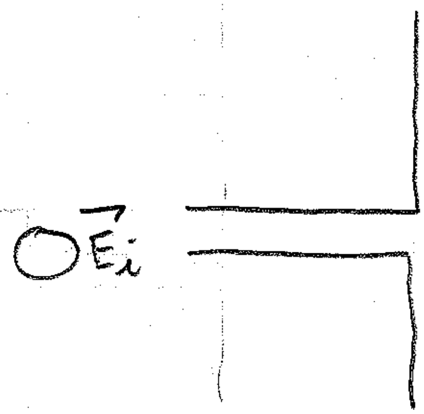
$$\vec{A} = \frac{1}{4\pi} \iint \frac{\vec{J} e^{ik|r-r'|}}{|r-r'|}$$

$$\vec{E} = -\frac{1}{i\omega\varepsilon} \vec{\nabla} \times \vec{\nabla} \times \vec{A}$$

$$P = \iiint \vec{E} \cdot \vec{J} \, d\vec{v}$$

But $E = 0$ in a conductor!

Think of an antenna this way



$$E_T = E_i + E_s$$

total incident scattered

We usually know the incident field

We know the boundaries

We want to find the scattered field

On the boundary $\vec{E}_s = -\vec{E}_t$

<u>An approximation is</u>

Equivalent Current <u>Induced</u>

$$\vec{A}_s = \frac{1}{4\pi} \iint \frac{\vec{J}_{ic}\, e^{ik|r-r'|}}{|r-r'|}$$

An antenna is a <u>scatterer</u>
not a radiator !

We should think of most E & M problems
as Sources fields & Scattered fields

Basic E&M Concepts

1) Maxwell's Equations
2) Circuit Equivalents
3) Sources
   - Electric (Current)
   - Magnetic (Voltage)
4) Energy & Power
   - Real
   - Reactive
5) Time Domain & Frequency Domain
6) Wave Solutions
7) Duality
8) Uniqueness
9) Image Theory
10) Equivalence
11) Induction
12) Reciprocity

Maxwells Equations

$$\vec{\nabla} \times \vec{\mathcal{E}} = -\frac{\partial \vec{\mathcal{B}}}{\partial t}$$

$$\vec{\nabla} \times \vec{\mathcal{H}} = \frac{\partial \vec{\mathcal{D}}}{\partial t} + \vec{\mathcal{J}}$$

$$\vec{\nabla} \cdot \vec{\mathcal{D}} = \rho$$

$$\vec{\nabla} \cdot \vec{\mathcal{B}} = 0$$

Cicuit Equations

$v = \int \vec{\mathcal{E}} \cdot d\vec{l}$ voltage (Volts)

$u = \int \vec{\mathcal{H}} \cdot d\vec{l}$ magnetomotive force (Amps

$\psi^e = \iint \vec{\mathcal{D}} \cdot d\vec{s}$ electric flux

$\psi^m = \iint \vec{\mathcal{B}} \cdot d\vec{s}$ magnetic flux

$q = \iiint \rho \, dv$ electric charge

$i = \iint \vec{\mathcal{J}} \cdot d\vec{s}$ electric current

$$\Sigma v = -\frac{\partial \psi^m}{\partial t}$$

$$\Sigma u = \frac{\partial \psi^e}{\partial t} + i$$

$$\Sigma \psi^e = q$$

$$\Sigma \psi^m = 0$$

Sources

$$\vec{\nabla} \times \vec{\mathcal{E}} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{\mathcal{H}} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\vec{J} = \sigma \vec{\mathcal{E}} + \vec{J}^i$$

$$i^i = \iint \vec{J}^i \cdot d\vec{s}$$

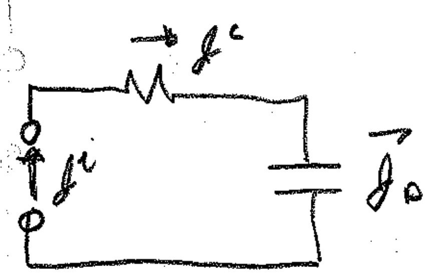
Where's the concept of a voltage source?

A current source can be thought of as a spatial change (or discontinuity) in the magnetic field $\mathcal{H}$.

$$\vec{J}^{+} = \frac{\partial \vec{D}}{\partial t} + \sigma \vec{E} + \vec{J}^i$$

$$J^{+} = \vec{J}_d + \vec{J}_c + \vec{J}^i$$

$$\nabla \times \vec{H} = \vec{J}^{+}$$

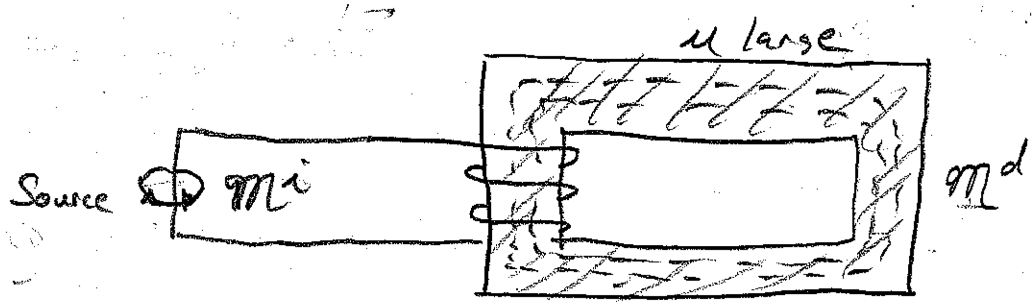


Voltage Sources

$$\vec{\nabla} \times \vec{E} = -\vec{\mathcal{M}}^{+}$$

$$\vec{\mathcal{M}}^{+} = \frac{\partial \vec{B}}{\partial t} + \mathcal{M}^{i}$$

$$= \mathcal{M}^{d} + \mathcal{M}^{i}$$



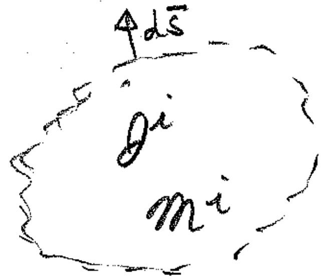
$$\vec{\nabla} \times \vec{\mathcal{H}} = \vec{J}^{+} \qquad \vec{\nabla} \times \vec{\mathcal{E}} = -\mathcal{M}^{+}$$

$$\oint \vec{\mathcal{H}} \cdot d\vec{l} = \iint \vec{J}^{+} \cdot d\vec{s} \qquad \oint \vec{\mathcal{E}} \cdot d\vec{l} = -\int \vec{\mathcal{M}}^{+} \cdot d\vec{s}$$

$$\oint \vec{\mathcal{H}} \cdot d\vec{l} = i^{+} \qquad \oint \vec{\mathcal{E}} \cdot d\vec{l} = -k^{+}$$

$$\Sigma u = i^{+} \qquad \Sigma v = -k^{+}$$

Energy & Power.



Poynting vector

$$\vec{S} = \vec{\mathcal{E}} \times \vec{\mathcal{H}}$$

Density power flux.

$$p_f = \vec{\nabla}\cdot\vec{S}$$

$$P_f = \iiint p_f \, dv$$

$$= \oiint \vec{S}\cdot d\vec{s} = \oiint \vec{\mathcal{E}}\times\vec{\mathcal{H}}\cdot d\vec{s}$$

$$\nabla\cdot \mathcal{E}\times\vec{\mathcal{H}} = \vec{\mathcal{H}}\cdot\vec{\nabla}\times\vec{\mathcal{E}} - \vec{\mathcal{E}}\cdot\nabla\times\vec{\mathcal{H}}$$

$$= -\vec{\mathcal{H}}\cdot\vec{\mathcal{M}}^t - \vec{\mathcal{E}}\cdot\vec{\mathcal{J}}^t$$

$$\oiint \vec{\mathcal{E}}\times\vec{\mathcal{H}}\, d\vec{s} + \iiint_V \mathcal{H}\cdot\mathcal{M}^t \, dv + \iiint_V \mathcal{E}\cdot\mathcal{J}^t \, dv = 0$$

$$\mathcal{E}\cdot\mathcal{J}^t = \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon\mathcal{E}^2\right) + \sigma\mathcal{E}^2 + \vec{\mathcal{E}}\cdot\vec{\mathcal{J}}^i$$

$$\mathcal{H}\cdot\mathcal{M}^t = \frac{\partial}{\partial t}\left(\frac{1}{2} \mu\mathcal{H}^2\right) + \vec{\mathcal{H}}\cdot\vec{\mathcal{M}}^i$$

Define Stored Electric Energy

$$W_e = \frac{1}{2}\iiint \varepsilon\mathcal{E}^2 \, dv$$

Magnetic Stored Energy.

$$W_m = \frac{1}{2} \iiint \mu \mathcal{H}^2 \, dv$$

Dissapated Power

$$\mathcal{P}_d = \iiint \sigma \mathcal{E}^2 \, dv$$

Supplied Power

$$\mathcal{P}_s = -\iiint \left(\vec{\mathcal{E}} \cdot \mathcal{J}^i + \vec{\mathcal{H}} \cdot \vec{\mathcal{M}}^i \right) dv$$

$$\mathcal{P}_s = \mathcal{P}_{flux} + \mathcal{P}_d + \frac{d}{dt} \left(W_e + W_m \right)$$

Time Domain — Frequency Domain.

We live in the time domain. Eventually all solutions must be time Domain. However solving in frequency domain is algebra instead of convolution.

$$f(t) = \frac{1}{2\pi} \int F(\omega)\, e^{i\omega t}\, d\omega$$

$$\frac{d\, f(t)}{dt} = \frac{1}{2\pi} \int i\omega\, F(\omega)\, e^{i\omega t}\, d\omega$$

Does the choice of $i\omega$ or the $\frac{1}{2\pi}$ make a difference

No but all test equipment uses this choice

$F(\omega)$ has units of Hz^{-1}

$$F(\omega) = \int f(t)\, e^{-i\omega t}\, d\omega$$

If $f(t)$ is real

$$F(-\omega) = F(\omega)^*$$

$$f(t) = \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} \, d\omega$$

$$= 2 \operatorname{Re} \left\{ \frac{1}{2\pi} \int_0^{\infty} F(\omega) e^{i\omega t} \, d\omega \right\}$$

Maxwell's Eqns in the freq. domain

$$\nabla \times \vec{E} = -i\omega \vec{B}$$

$$\nabla \times \vec{H} = i\omega \vec{D} + \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \rho$$

<u>Sources</u>

$$\nabla \times \vec{E} = -\vec{M}^+$$

$$\nabla \times \vec{H} = \vec{J}^+$$

<u>Induced currents</u>

$$\vec{J} = (\hat{\sigma} + i\omega \hat{\varepsilon}) \vec{E} = \hat{y}(\omega) \vec{E}$$

$$\vec{M} = i\omega \hat{\mu} \vec{H} = \hat{z}(\omega) \vec{H}$$

$$-\nabla \times \vec{E} = \hat{z}(\omega)\,\vec{H} + \vec{M}^{i}$$

$$\nabla \times \vec{H} = \hat{y}(\omega)\,\vec{E} + J^{i}$$

where
$$\hat{z}(\omega) = i\omega\hat{\mu}(\omega)$$

$$\hat{y}(\omega) = i\omega\,\hat{\varepsilon}(\omega)$$

Complex Power

$$a = |A| \cos(\omega t + \alpha) = \mathrm{Re}\,(A e^{i\omega t})$$

$$\mathcal{B} = |B| \cos(\omega t + \beta) = \mathrm{Re}\,(B e^{i\omega t})$$

$$\langle a\mathcal{B} \rangle_{time} = \frac{1}{2} |A||B| \cos(\alpha - \beta)$$

$$= \frac{1}{2} \mathrm{Re}\,(A B^{*})$$

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^{*}$$

$$\langle \vec{\mathcal{S}} \rangle = \mathrm{Re}\,(\vec{S})$$

$$\langle \mathcal{P}_d \rangle = \frac{1}{2} \iiint \sigma |E|^2 \, dv$$

$$\langle W_e \rangle = \frac{1}{4} \iiint \varepsilon |E|^2 \, dv$$

$$\langle W_m \rangle = \frac{1}{4} \iiint \mu |H^2| \, dv$$

Complex Power Equation

$$\hat{P}_f = \frac{1}{2}\oiint \vec{E}\times\vec{H}^{*}\cdot d\vec{s}$$

complex number.

$$\hat{P}_s = -\frac{1}{2}\iiint\left(\vec{E}\cdot\vec{J}^{i*} + \vec{H}\cdot\vec{M}^{i*}\right)dv$$

complex number

$$\langle P_s\rangle = Re\left(\hat{P}_s\right)$$

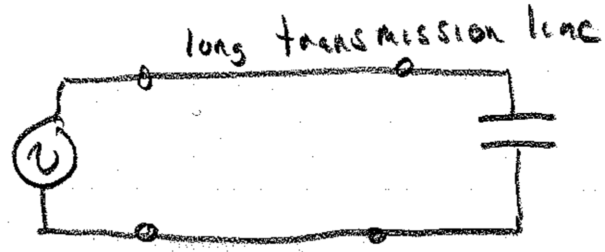
$$\hat{P}_s = \hat{P}_f + \langle P_d\rangle + i2\omega\left(\langle W_m\rangle - \langle W_e\rangle\right)$$

due to complex algebra

Reactive Power

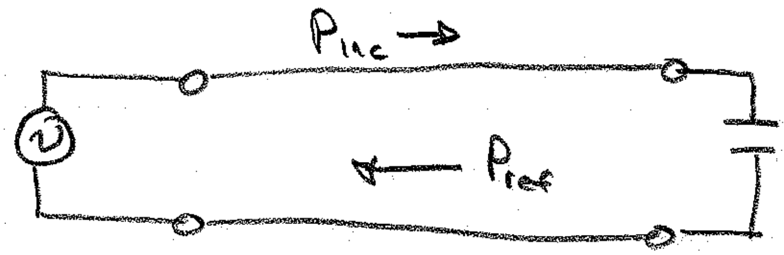
What is reactive Power ?

Consider



The time average power dissapated into the capacitor is zero.

But we can think of incident power travelling down the line charging the capacitor and reflected power traveling back.



The stored energy sloshing back & forth out of the capacitor can be thought of reactive Power.

Why do we care about reactive power?

Reactive power is associated with out of phase currents & voltages. The structure must withstand these out of phase voltages & currents

Wave Solutions

Consider

$$\vec{\nabla} \times \vec{E} = -\hat{z}\,\vec{H}$$

$$\vec{\nabla} \times \vec{H} = \hat{y}\,\vec{E} + \vec{J}$$

Because $\nabla \cdot \vec{H} = 0$

We can have $\nabla \times A = \vec{H}$

$$\nabla \times \vec{E} = -\hat{z}(\nabla \times A)$$

$$\nabla \times (E + \hat{z}\,A) = 0$$

Since $\nabla \times (\nabla \phi) = 0$ (Identity)

$$E + \hat{z}\,A = -\nabla \phi$$ electric scalar potential

ie for $\omega \to 0$ $\hat{z} \to 0$

$$\vec{E} = -\nabla \phi$$

$$\nabla \times \vec{H} = \hat{y}\,\vec{E} + \vec{J}$$

$$\nabla \times (\nabla \times A) = \hat{y}(-\nabla\phi - \hat{z}\,A) + \vec{J}$$

$$\nabla \times (\nabla \times \vec{A}) + \hat{y}\hat{z}\,\vec{A} + \hat{y}\,\nabla\phi = \vec{J}$$

$$\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} + \hat{y}\hat{z}\,\vec{A} + \hat{y}\,\nabla\phi = \vec{J}$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A} + \hat{y}\phi) = \nabla^2\vec{A} - \hat{y}\hat{z}\vec{A} + \vec{J}$$

Since we only chose $\nabla \times \vec{A} = H$

we are still free to chose $\nabla \cdot \vec{A}$ (our gauge)

<u>We choose</u>

$$\vec{\nabla}\cdot\vec{A} + \hat{y}\phi = 0$$

$$\nabla^2\vec{A} - \hat{y}\hat{z}\vec{A} = -\vec{J}$$

$$\vec{H} = \nabla \times \vec{A}$$

$$\vec{E} = -\hat{z}\vec{A} + \frac{1}{\hat{y}}\nabla(\vec{\nabla}\cdot\vec{A})$$

<u>Free space Green's function</u>

Let $\vec{J} = I\vec{l}\,\delta(\vec{r}-\vec{r}')$

$$= I\vec{l}\,\delta(x-x')\,\delta(y-y')\,\delta(z-z') \text{ (Units)}$$

Let $k^2 = -\hat{y}\hat{z}$

$$A = \frac{I\vec{l}}{4\pi}\frac{e^{-k|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$$

In General, In free Space

$$\vec{A}(\vec{r}) = \frac{1}{4\pi} \iiint \frac{J(r')\, e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\, dv$$

Now Consider

$$\nabla \times E = -\hat{z}\,\vec{H} - \vec{M}$$

$$\nabla \times \vec{H} = \hat{y}\,\vec{E}$$

Charge free region

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\therefore \quad \vec{E} = -\vec{\nabla} \times \vec{F}$$

$$\nabla \times \vec{H} = -\hat{y}(\nabla \times \vec{F})$$

$$\nabla \times (H + \hat{y}\vec{F}) = 0$$

$$\vec{H} + \hat{y}\vec{F} = -\nabla \phi_m$$

$$\nabla \times \vec{E} = -\hat{z}\,\vec{H} - \vec{M}$$

$$-\nabla \times \nabla \times \vec{F} = +\hat{z}\left(\vec{\nabla}\phi_m + \hat{y}\vec{F}\right) - \vec{M}$$

$$-\nabla(\nabla \cdot \vec{F}) + \nabla^2 \vec{F} = \hat{z}\,\nabla\phi_m + \hat{y}\hat{z}\vec{F} - \vec{M}$$

$$-\nabla\left(\vec{\nabla} \cdot \vec{F} + \hat{z}\,\phi_m\right) = -\nabla^2\vec{F} + \hat{y}\hat{z}\,\vec{F} - \vec{M}$$

We can choose

$$\vec{\nabla}\cdot\vec{F} + \hat{z}\,\phi_m = 0$$

$$\therefore \quad \nabla^2\vec{F} - \hat{y}\hat{z}\,\vec{F} = -\vec{M}$$

$$E = -\nabla\times\vec{F}$$

$$\vec{H} = -\hat{y}\,\vec{F} + \frac{1}{\hat{z}}\,\vec{\nabla}(\vec{\nabla}\cdot\vec{F})$$

<u>Free Space Green's Function</u>

$$\vec{M} = V\vec{l}\,\delta(\vec{r}-\vec{r}\,')$$

$$= V\vec{l}\,\delta(x-x')\,\delta(y-y')\,\delta(z-z')$$

$$\vec{F} = \frac{V\vec{l}}{4\pi}\,\frac{e^{-k|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}$$

In General, in free space

$$\vec{F}(\vec{r}) = \frac{1}{4\pi}\iiint M(\vec{r}\,')\,\frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\,dv$$