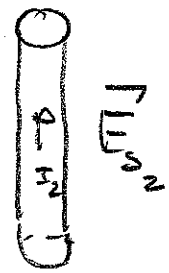
Antenna Impedance

Induced EMF method



Total Complex Power

$$S = \frac{1}{2} E_s \times H_s^*$$

scattered

$$P_{complex} = \frac{1}{2} \int_{\substack{\text{Cylinder}\\\text{Surface}}} \vec{E}_s \times \vec{H}^* \cdot d\vec{a}$$

$$= \frac{1}{2} \int_{-L}^{L} E_z \Big|_{\rho=\rho_\delta} H_\varphi^* \Big|_{\rho=\rho_\delta} 2\pi \rho_\delta \, dl$$

ρ_δ = radians of wire

$$H_\varphi \Big|_{\rho=\rho_\delta} \cong \frac{I_z}{2\pi\rho_\delta}$$

$$P_{complex} = \frac{1}{2} \int_{-L/2}^{L/2} E_z \Big|_{\rho=\rho_\delta} I_z^* \, dl$$

$$I = I_0 \frac{f(z)}{f(0)}$$

I_0 = current at driving point

$$P_{complex} = \frac{1}{2} I_0^* \int_{-L/2}^{L/2} E_z \Big|_{\rho=\rho_0} \frac{f^*(z)}{f^*(0)} \, dz$$

$$P_{complex} = \frac{1}{2} I_0^* I_0 Z_{11}$$

$$Z_{11} = R_{11} + j X_{11} = \frac{1}{I} \int_{-\frac{L}{2}}^{L/2} E_z \Big|_{\rho=\rho_0} \frac{f^*(z)}{f(0)} \, dz$$

<u>But</u>

$$A_z = \frac{\mu_0 I_0}{4\pi} \int_{-L}^{L} \frac{f^*(z)}{f(0)} \frac{e^{-jk|r-r'|}}{|r-r'|} \, dz'$$

Can't use far field appox.

$$\vec{H} = \vec{\nabla} \times \vec{A}_z$$

$$E = \frac{1}{j\omega\varepsilon_0} \vec{\nabla} \times \vec{H}$$

you do the math!

Half wave dipole

$$f(z) = \cos kz$$

big assumption.

$$\frac{kL}{2} = \frac{\pi}{2}$$

$$Z_{11} = 73 + j\,42.5 \ \Omega$$

For a pure $\frac{1}{2}$ wave dipole, this result is independent of wire diameter.

However for lengths different from $\frac{\lambda}{2}$, the imaginary part of the impedance is a function of wire width and can be tuned out.

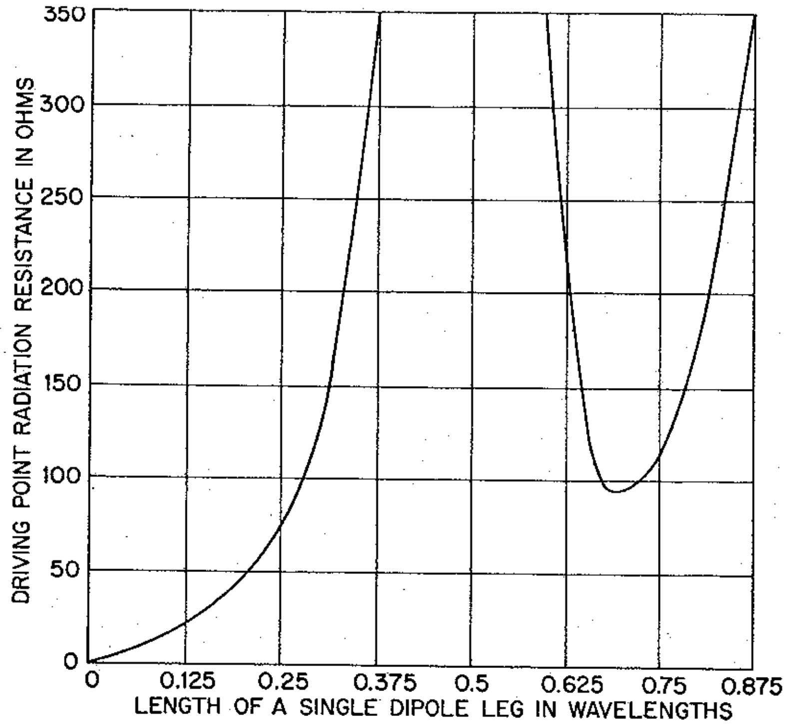
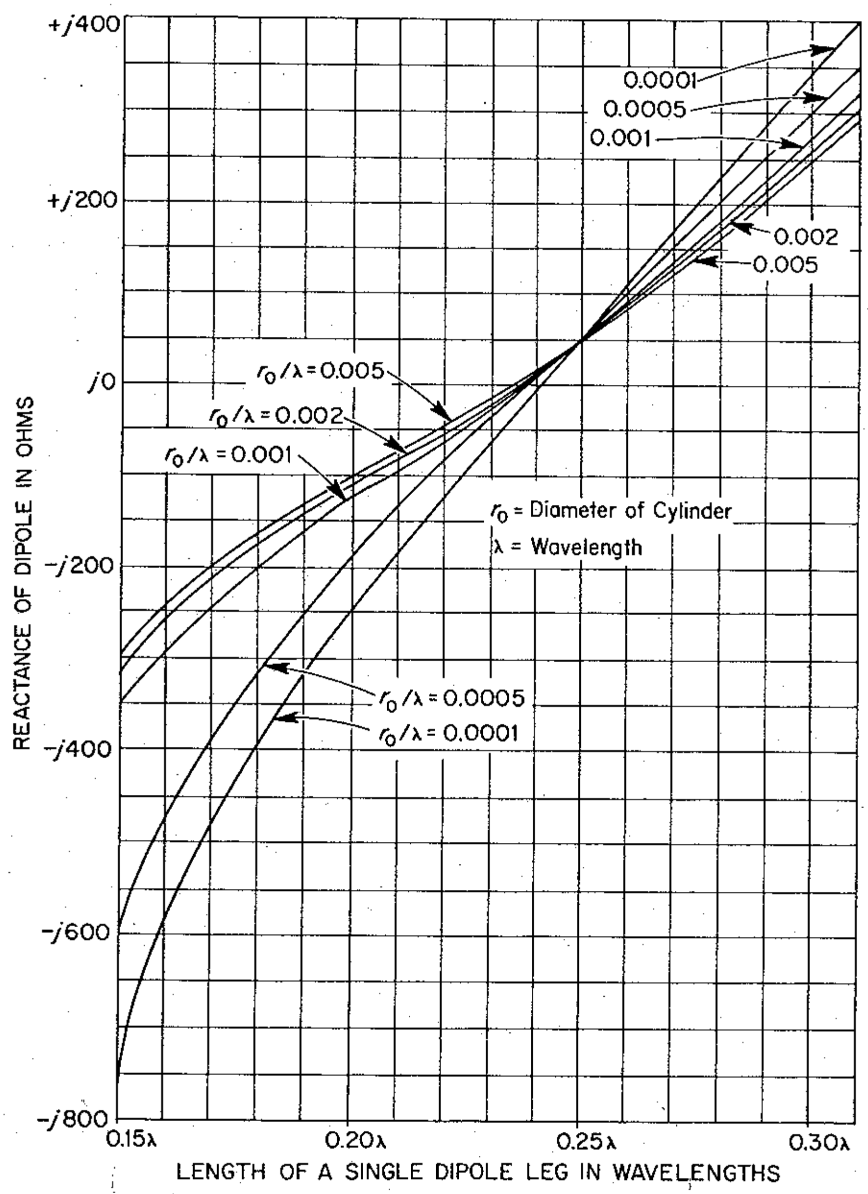
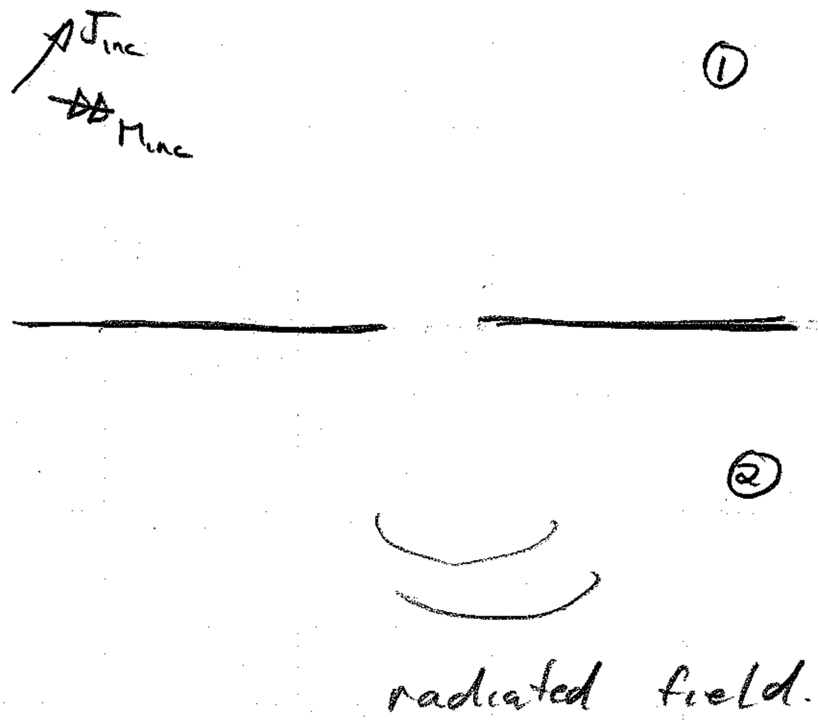


Fig. 13–9. Dipole radiation resistance referred to driving point.

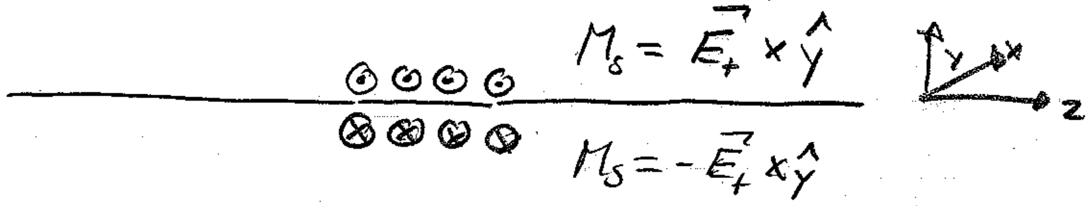


Moment Methods

Switch Gears & look at slots in ground planes



Becomes



$$M_s = \vec{E}_t \times \hat{y}$$

$$M_s = -\vec{E}_t \times \hat{y}$$

But what is $\vec{E}_t$?

$t \Rightarrow$ tangential field on conductor surface

The magnetic surface currents guarantee that the tangential electric field is continuous thru the slot.

We have to make sure that the tangental magnetic fields are also continuous thru the slot.

$$\vec{H}_t^{(1)} = \vec{H}_t^{(inc)} + \vec{H}_t^{(1)}(\vec{E}_t \times \hat{y})$$
$$\vec{H}_2^{(2)} = H_t^{(2)}(-\vec{E}_t \times \hat{y})$$

$$-H_t^{(inc)} = H_t^{(1)}(\vec{E}_t \times \hat{y}) + H_t^{(2)}(\vec{E}_t \times \hat{y}) \quad * $$

Guess $\vec{E}_t$ of the form

$$\vec{E}_t = \hat{x}\sum_n E_{x_n}\theta_n(x,z) + \hat{z}\sum_n E_{z_n}\psi_n(x,z)$$

Where θ_n & ψ_n are complete orthoganal sets of functions

Let ϕ_m be another complete set of weighting functions

Green's Functions

$$(\nabla^2 + k^2)\, G(\vec{r}|\vec{r}\,') = -\delta(\vec{r}-\vec{r}\,')$$

$$F(\vec{r}) = \iiint_{V'} M(\vec{r}\,')\, G(\vec{r}|\vec{r}\,')\, dv'$$

$$\vec{H}(\vec{r}) = -\frac{j\omega}{k^2}\left(k^2 \iint_S M_s(\vec{r}\,')\, G(\vec{r}|\vec{r}\,')\, ds' \right.$$

$$\left. + \nabla \iint_S (\nabla' \cdot M_s(\vec{r}\,')\, G(\vec{r}|\vec{r}\,')\, ds' \right)$$

$$\vec{H}^{(k)}(\vec{r}) = \vec{\mathcal{H}}^{(k)}(\vec{M}_s)$$

Define the following

$$\langle \phi_m | \mathcal{H}_v^{(k)} | \Theta_n \rangle = \iint_{x,z} \phi_m \mathcal{H}_v^{(k)} (\hat{z}\Theta_n)\, da$$

$$\langle \phi_m | \mathcal{H}_v^{(k)} | \Psi_n \rangle = \iint_{x,z} \phi_m \mathcal{H}_v^{(k)} (\hat{x}\, \Psi_n)\, da$$

$$\langle \phi_m | H_v^{(inc)} \rangle = \iint_{x,z} \phi_m H_v^{(inc)}\, da$$

$$-\langle \phi_m | H_x^{inc} \rangle = \sum_n \left(\sum_k \langle \phi_m | \mathcal{H}_x^{(k)} | \Psi_n \rangle \right) E_{zn} + \sum_n \left(\sum_k \langle \phi_m | \mathcal{H}_x^{(k)} | \Theta_n \rangle \right) E_{xn}$$

$$-\langle \phi_m | H_z^{inc} \rangle = \sum_n \left(\sum_k \langle \phi_m | \mathcal{H}_z^{(k)} | \Psi_n \rangle \right) E_{zn} + \sum_n \left(\sum_k \langle \phi_m | \mathcal{H}_z^{(k)} | \Theta_n \rangle \right) E_{xn}$$

For a rectangular skinny slot with the length along the x direction

$$-\langle \phi_m | H_x^{inc} \rangle \cong \sum_n \left(\sum_k \langle \phi_m | \mathcal{H}_x^{(k)} | \Psi_n \rangle \right) E_{zn}$$