FERMILAB
# ENGINEERING NOTE

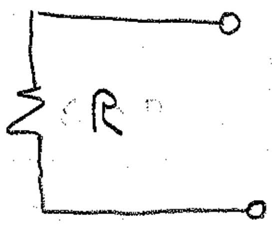
| SECTION | PROJECT | SERIAL-CATEGORY | PAGE |
|---|---|---|---|
| | | | 1 |

SUBJECT: Noise.

NAME:

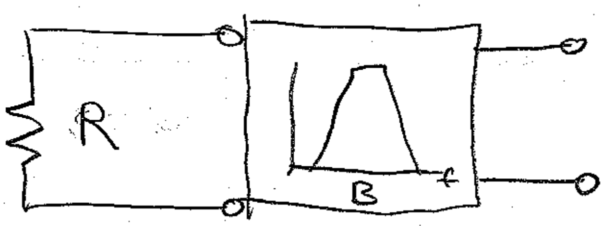
DATE: | REVISION DATE:

Consider a Resistor at a temperature T



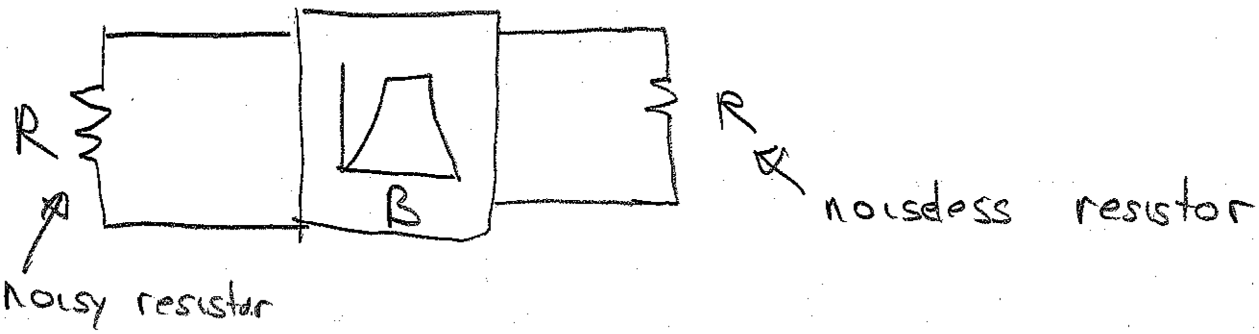

This resistor produces noise over a spectrum.

Place a filter next to the noise source




The filter has a bandwidth B.

To collect the maximum noise from the resistor, we must match the load impedance to the source impedance



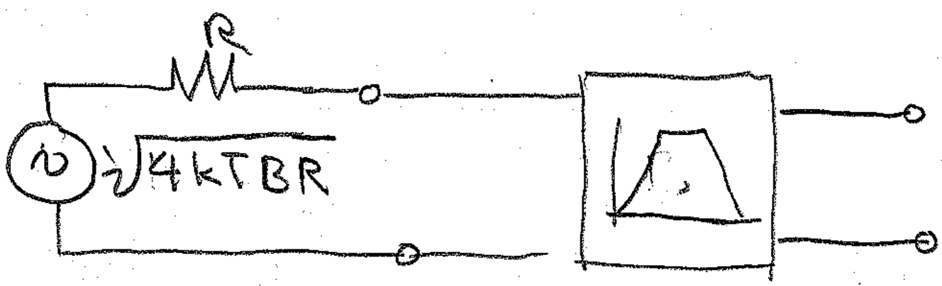

From Thermodynamics the <u>average</u> power <u>available</u> to the noiseless resistor

is $\langle P_{av} \rangle = kTB$

$k = 13.6 \times 10^{-24}$ W/Kelvin/Hz

or $k = -174$ dBm/Hz at $T = 293$K

An equivalent model for the noisy resistor is



FERMILAB
ENGINEERING NOTE

SECTION | PROJECT | SERIAL-CATEGORY | PAGE 3.

SUBJECT | NAME | DATE | REVISION DATE

Why not just make R small and have no noise power?

Because to match the source to get the maximum power transfer, the load resistor would have to be made small so you can't win.

The available power density from a resistor is

$$S(f) = kT$$

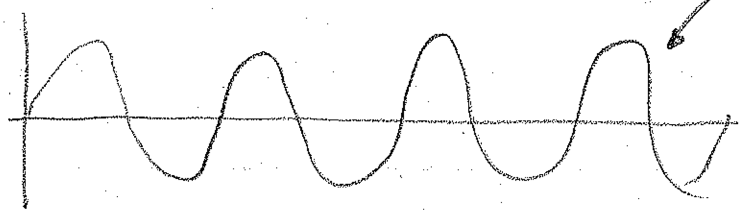
No matter what the value of the resistance is!

FERMILAB ENGINEERING NOTE

SECTION | PROJECT | SERIAL-CATEGORY | PAGE 4

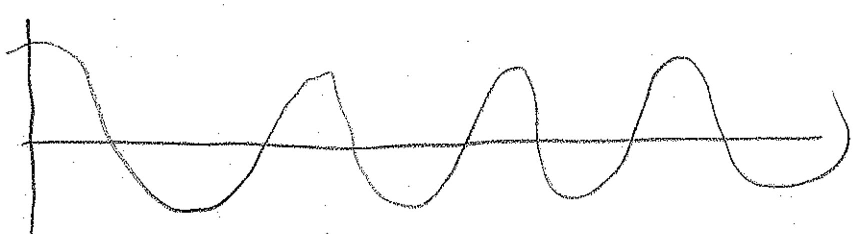
SUBJECT | NAME | DATE | REVISION DATE

Consider a very small bandwidth $\Delta B$ a make a single measurement of the voltage deposited in the load resistor.



looks like a sine wave because of the narrow filter.

Do another measurement



It has a different phase because its random.

The voltage on a given measurement is

$$V = \mathrm{Re}\left\{ V_m e^{j\varphi_m} e^{j\omega t} \right\}$$

$$V_m e^{j\varphi_m} = V_{rm} + j V_{im}$$

The average value of the real or imaginary voltage is zero.

$$\langle V_r \rangle = \frac{1}{M}\sum_m V_{rm} = 0$$

$$\langle V_i \rangle = \frac{1}{M}\sum_m V_{im} = 0$$

The RMS is not zero

Since

$$\frac{\langle V^2 \rangle}{R} = \frac{\langle V_r^2 \rangle}{R} + \frac{\langle V_i^2 \rangle}{R} = kTB$$

Then

$$\frac{\langle V_r^2 \rangle}{R} = \frac{1}{R}\frac{1}{M}\sum V_r^2 = \frac{kTB}{2}$$

$$\frac{\langle V_i^2 \rangle}{R} = \frac{1}{R}\frac{1}{M}\sum V_i^2 = \frac{kTB}{2}$$

Define $p_r^2 = \frac{V_r^2}{R}$ $\quad \langle p_r^2 \rangle = \frac{kTB}{2}$

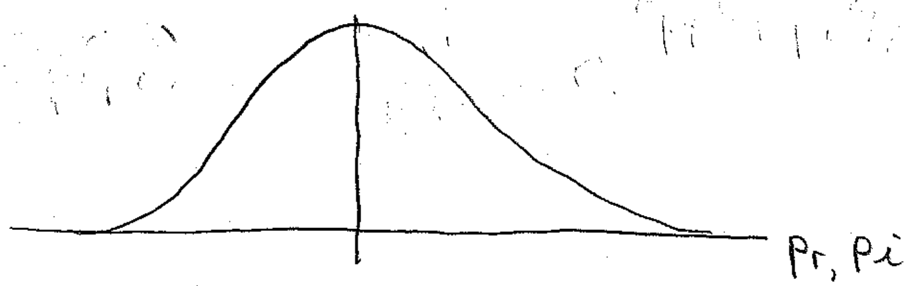
$p_i^2 = \frac{V_i^2}{R}$ $\quad \langle p_i^2 \rangle = \frac{kTB}{2}$

$$\langle p_r^2 \rangle + \langle p_i^2 \rangle = \langle P \rangle = kTB$$

| FERMILAB ENGINEERING NOTE | SECTION | PROJECT | SERIAL-CATEGORY | PAGE 6 |
|---|---|---|---|---|
| SUBJECT | | | NAME | |
| | | | DATE | REVISION DATE |

Probability of a measurement

$$\Psi(p) = \frac{1}{\sqrt{2\pi \frac{kTB}{2}}} e^{-\frac{1}{2} p^2 / \frac{kTB}{2}}$$

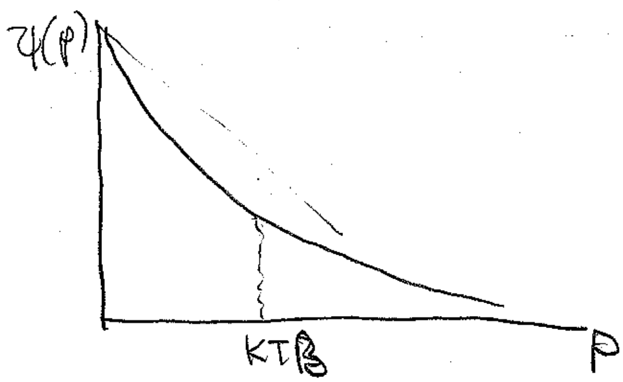



$$\Psi(P) = \Psi(p_r)\Psi(p_i)$$

$$= \frac{1}{\pi kTB} e^{-\frac{(p_r^2 + p_i^2)}{kTB}} = \frac{1}{\pi kTB} e^{-\frac{P^2}{kTB}}$$

$$\langle P \rangle = kTB$$

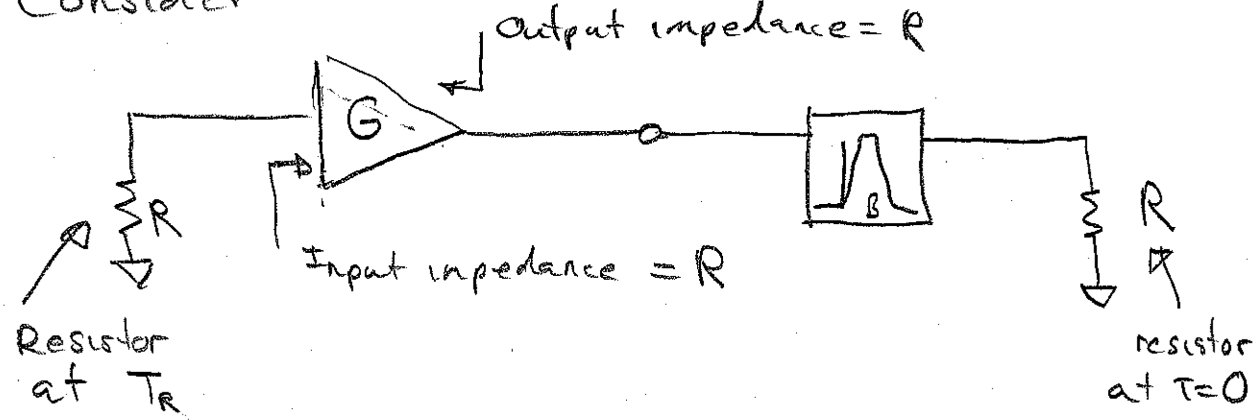
$$\langle P^2 \rangle = (kTB)^2$$




1) Most like value is 0
2) Average value is kTB
3) RMS value is kTB

# Noise in Amplifiers

Consider




G = power Gain

What is the power recieved into the noiseless resistor?

If the amplifier is noiseless then:

$$P_r = G k T_R B$$

If the amplifier has noise, we always, always, define the noise measured on the output; refered to the input

$$P_A = G k T_A B$$

measured on output

refered to input

$T_A$ is not an actual temperature, but an effective temperature.

The noise figure is defined as.

$$N_f = \frac{P_A + P_R\big|_{293K}}{P_R\big|_{293K}}$$

The noise figure is typically refered to in dB (because it is a power ratio)

The noise figure can never be lower than 0 dB.

$$P_A = P_R\big|_{293K} (N_f - 1)$$

$$T_A = 293K (N_f - 1)$$

If $N_f = 2$ (3dB)

$T_A = 293K$

$I_f \quad N_f = .7 \text{ dB}$

$N_F = 1.175$

$T_A = 51 \text{ K}$

Example,

The noise floor of a spectrum analyzer measured with its input terminated in $50\,\Omega$ is $-90$ dBm. The resolution bandwidth is 1 MHz.

a) What is the noise figure of the SA

b) What is the noise Temp of the SA

$$P_m = -90 \text{ dBm into } 1 \text{ MHz}$$

$$\therefore P_m = (-90-60) \text{ dBm} = -150 \text{ dBm into } 1 \text{ Hz}$$

$$-60 = 10 \log_{10}\left(\frac{1 \text{ Hz}}{1 \text{ MHz}}\right)$$

$$P_R\Big|_{293} = -174 \text{ dBm into } 1 \text{ Hz}$$

$$P_m = P_{SA} + P_R\Big|_{293K}$$

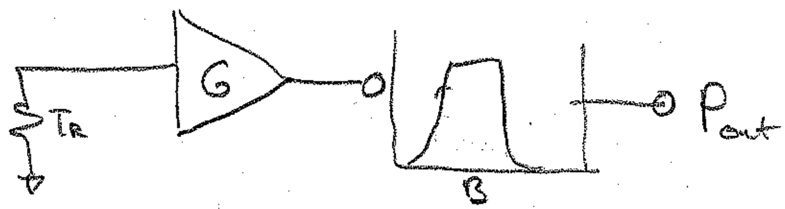
$$N_F = P_m / P_R\Big|_{293K}$$

$$N_{F_{dB}} = -150 \text{ dBm} - (-174 \text{ dBm}) = 24 \text{ dBm}$$

$$T_{SA} = 73,300 \text{ K} \;!!$$

# Measuring Noise Figure

## Simple (Error prone way)




Measure G with a network analyzer

Measure $P_{out}$ with a spectrum analyzer

$$P_o = G k T_R B + G k T_A B$$

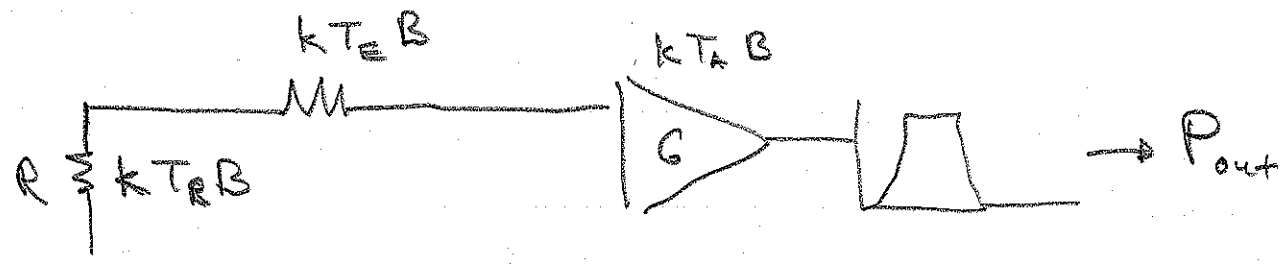
$$\frac{P_o}{G} = k T_R B + k T_A B$$

$$\frac{P_{out}}{G k T_R B} = \frac{k T_R B + k T_A B}{k T_R B} = N_f$$

Need to know G precisely

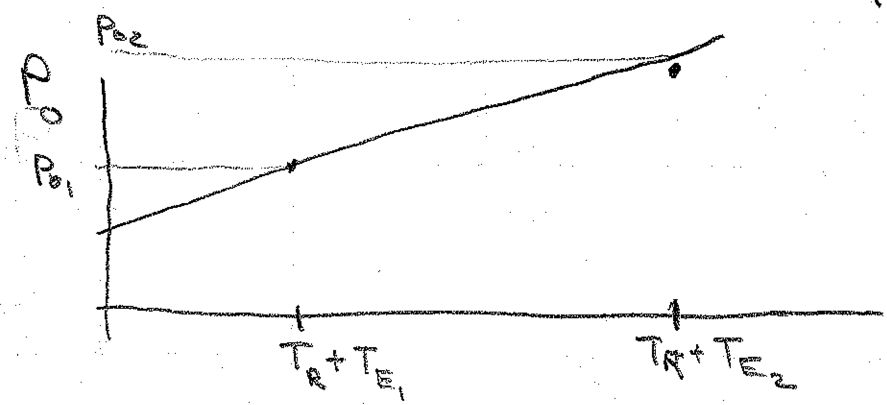
Need to know B precisely.

2nd Technique (Y factor)




Assume I can change $T_E$

Make Two measurements $T_{E_1}$ & $T_{E_2}$




Find the intercept = Power added by Amp

$$P_o = \frac{P_{o_2} - P_{o1}}{T_{E_2} - T_{E_1}} \left(T - T_R - T_{E_1}\right) + P_{o_1}$$

$$P_{int} = P_{o1} - \frac{(T_R + T_{E_1})}{(T_{E_2} - T_{E_1})}(P_{o2} - P_{o1})$$

$$T_1 = T_r + T_{E_1} \qquad T_2 = T_r + T_{E_2}$$

$$T_r = T_1 - T_{E_1}$$

$$P_o = \frac{\Delta P}{\Delta T}(T - T_1) + P_{o1}$$

$$N_f = \frac{P_o\big|_{T=T_r}}{P_o\big|_{T=T_r} - P\big|_{T=0}}$$

$$P_o\big|_{T=T_R} = P_{o1} - \frac{\Delta P\, T_{E_1}}{\Delta T}$$

$$P_o\big|_{T=0} = P_{o1} - \Delta P \frac{T_1}{\Delta T}$$

$$P_o\big|_{T=R} - P_o\big|_{T=0} = \frac{\Delta P}{\Delta T}(T_1 - T_{E_1})$$

$$N_f = \frac{P_{o1} - \frac{\Delta P}{\Delta T} T_{E_1}}{\frac{\Delta P}{\Delta T}(T_1 - T_{E_1})}$$

$$N_f = \frac{P_{o1}}{\Delta P} \frac{\Delta T}{(T_1 - T_{E_1})} - \frac{T_{E_1}}{(T_1 - T_{E_1})}$$

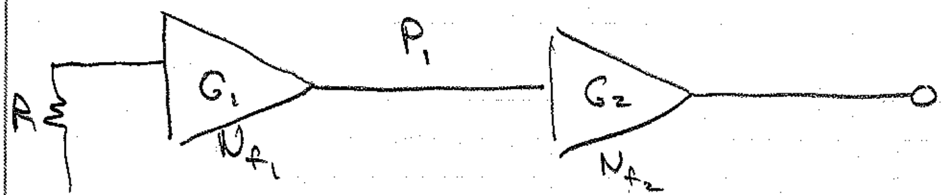
For $T_{E_1}$ small

$$N_f = \frac{P_{o1}}{\Delta P} \frac{\Delta T}{T_r}$$

# Noise figure of Systems

$$N_f = \frac{P_R + P_A}{P_R}$$

$$(N_f - 1) P_R = P_A$$




$$P_1 = G_1 (P_R + P_{A_1})$$

$$P_2 = G_1 G_2 (P_R + P_{A_1}) + G_2 P_{A_2}$$

$$P_2 = G_1 G_2 P_R + G_1 G_2 (N_{f_1} - 1) P_R + G_2 (N_{f_2} - 1) P_R$$

$$N_{f_{Total}} = \frac{P_2 / G_1 G_2}{P_R}$$

$$N_{f_{Total}} = N_{f_1} + \frac{N_{f_2} - 1}{G_1}$$

$$N_{f_{Total}} = N_{f_1} + \frac{N_{f_2} - 1}{G_1} + \frac{N_{f_3} - 1}{G_1 G_2}$$

Example

$G_1 = 20\ dB$

$N_{f1} = .7\ dB$

$G_2 = 20\ dB$

$N_{f2} = 3\ dB$

---

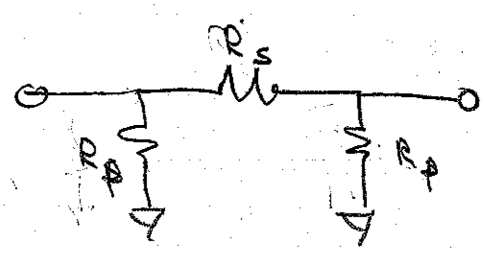
$N_{f2} = 2$

$G_1 = 10$

$N_{f\,total} = .7 + \frac{2-1}{10} = .8\,!$

---

Noise Figure of an Attenuator




for 6 dB

$R_s = 37\ \Omega$

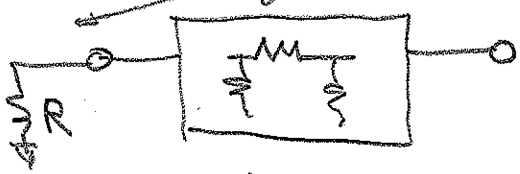
$R_p = 150\ \Omega$

for 20 dB

$R_s = 247\ \Omega$

$R_p = 61\ \Omega$

So an attenuator is just a resistor network all the same temperature




Attenuation = $A$

What is the power on the output

$$P_o = kT_R B$$

Refere this to the input

$$G = \frac{1}{A}$$

$$P_o \Big|_{input} = A\,kT_R B$$

$$N_f = \frac{A\,kT_R B}{kT_R B} = A$$

A 3dB attenuator has a noise figure of

3dB

Example


$$G_1 = 20 \text{ dB}$$

$$N_1 = .7 \text{ dB} \Rightarrow 1.175$$

$$N_{f_A} = 3\text{dB} \Rightarrow 2$$

$$G_{Atten} = 1/2$$

$$N_{f_{Tot}} = 2 + \frac{1.175-1}{1/2}$$

$$= 2 + .35$$

$$= 2.35$$

$$= 3.711 \text{ dB}$$

$$T_{eff} = 395 \text{ K}$$